ONLINE APPENDIX
for
“PUBLIC COMMUNICATION AND INFORMATION ACQUISITION”

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A Computing Equilibrium

A.1 Equilibrium Actions

In this section, I solve for the equilibrium coefficients of the agent's action rule, taking the information structure $k \leq n$ as given. I conjecture that the aggregate action rule takes the form given in equation (10), derive agent i's optimal response, and compute aggregate actions given the hypothesized rule. Equilibrium is a fixed point of the resulting mapping.

Throughout, I denote with a tilde any equilibrium objects taken as given by agent i. For example, since agent i takes aggregate actions as given, the aggregate action from the perspective of agent i is assumed to be [1]

$$\bar{p} = \widetilde{\psi_1} \sum_{l=1}^{n} g_l + \widetilde{\psi_2}\theta. \tag{A.1}$$

Let γ_1 and γ_2 be defined as in the text. Then, agent i's expectation of the state and average action are given respectively by

$$E(\theta|\mathcal{I}^i) = \gamma_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + \gamma_2 r^i \tag{A.2}$$

$$E(\bar{p}|\mathcal{I}^i) = \widetilde{\psi_1} \sum_{l=1}^{n} E(g_l|\mathcal{I}^i) + \widetilde{\psi_2} E(\theta|\mathcal{I}^i). \tag{A.3}$$

Since I assume that agents know the identity of the signals they have observed, the conditional expectation of signal l is given by

$$E\left(g_l|\mathcal{I}^i\right) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{A.4}$$

[1] Myatt and Wallace (2012) discuss the mild restrictions required to ensure the linear equilibrium is the unique equilibrium.

After some simplification, we can compute the expectation of the aggregate action

$$E(\bar{p}|\mathcal{I}^i) = \left(\widetilde{\psi_1}(1+(n-k)\gamma_1) + \widetilde{\psi_2}\gamma_1\right)\sum_{l=1}^{n}\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l + \gamma_2\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)r^i. \tag{A.5}$$

Evaluating the agent first order condition in expression (2), we get agent i's choice of action as a function of her observations:

$$\begin{aligned} p^i = &\left((1-\alpha)\gamma_1 + \alpha\left(\widetilde{\psi_1}(1+(n-k)\gamma_1)+\widetilde{\psi_2}\gamma_1\right)\right)\sum_{l=1}^{n}\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l \\ &+ \gamma_2\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right)r^i. \end{aligned} \tag{A.6}$$

Rearranging the weights on the public and private signals in equation (A.6), define

$$\psi_1^i \equiv \alpha\widetilde{\psi_1} + \gamma_1\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right) \tag{A.7}$$

$$\psi_2^i \equiv \gamma_2\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right) \tag{A.8}$$

to be the coefficients of agent i's optimal action rule, given (any) aggregate coefficients $\widetilde{\psi_1}$ and $\widetilde{\psi_2}$. In order to compute the average action, I must compute the cross-sectional average of $\mathbb{1}[g_l \in \mathcal{I}^i]g_l$.[2] By assumption, the set of signals observed is unrelated to the realizations of the signals themselves. Thus, this is just $E(\mathbb{1}[g_l \in \mathcal{I}^i])g_l = prob(g_l \in \mathcal{I}^i)g_l$. Because sampling is purely random, all possible combinations of signals observed are equally likely and we can immediately conclude that $prob(g_l \in \mathcal{I}^i) = \frac{k}{n}$.

Using this fact, I integrate equation (A.6) across agents to arrive at an expression for the aggregate action:

$$\begin{aligned} \bar{p} = &\frac{k}{n}\left((1-\alpha)\gamma_1 + \alpha\left(\widetilde{\psi_1}(1+(n-k)\gamma_1)+\widetilde{\psi_2}\gamma_1\right)\right)\sum_{l=1}^{n}g_l \\ &+ \gamma_2\left((1-\alpha)+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right)\theta. \end{aligned} \tag{A.9}$$

Comparing equations (A.1) and (A.9), I conclude that the equilibrium coefficient is a fixed point of the recursive relationship

$$\begin{bmatrix} \psi_1' \\ \psi_2' \end{bmatrix} = (1-\alpha)\begin{bmatrix} \frac{k}{n}\gamma_1 \\ \gamma_2 \end{bmatrix} + \alpha\begin{bmatrix} \frac{k}{n}(1+(n-k)\gamma_1) & \frac{k}{n}\gamma_1 \\ (n-k)\gamma_2 & \gamma_2 \end{bmatrix}\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}. \tag{A.10}$$

[2]See Judd (1985); Uhlig (1996) for a discussion of the issues related to using a law of large numbers when integrating across a continuum of agents.

Solving for the fixed point and substituting in for γ_1 and γ_2 yields the expressions

$$\psi_1^* = \left(n + \left(\frac{n}{k} - \alpha\right)\sigma_\eta^2 \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)\right)^{-1} \tag{A.11}$$

$$\psi_2^* = \left(1 + \sigma_\xi^2 \left(\frac{1}{1-\alpha} + \frac{k}{1-\alpha\frac{k}{n}}\frac{1}{\sigma_\eta^2}\right)\right)^{-1}. \tag{A.12}$$

A.2 Morris and Shin (2002) Effect

Under the assumption that $n = k = 1$ and $\alpha^\star = 0$, the equilibrium action coefficients are given by

$$\psi_1^* = \left(1 + \sigma_\eta^2 \left(1 + \frac{1-\alpha}{\sigma_\xi^2}\right)\right)^{-1} \tag{A.13}$$

$$\psi_2^* = \left(1 + \frac{\sigma_\xi^2}{1-\alpha}\left(1 + \frac{1}{\sigma_\eta^2}\right)\right)^{-1}. \tag{A.14}$$

Social losses are given by

$$-U^G = (\psi_1^* + \psi_2^* - 1)^2 + (\psi_1^*)^2\sigma_\eta^2 + (\psi_2^*)^2\sigma_\xi^2. \tag{A.15}$$

Taking the derivative with respect to σ_η^2 yields

$$\frac{\partial U^G}{\partial \sigma_\eta^2} = -\frac{(\sigma_\xi^2)^2\left(\sigma_\xi^2(1+\sigma_\eta^2) + (1-2\alpha)(1-\alpha)\sigma_\eta^2\right)}{\left(\sigma_\xi^2(1+\sigma_\eta^2) + (1-\alpha)\sigma_\eta^2\right)^3}, \tag{A.16}$$

which is greater than zero if any only if

$$(2\alpha - 1)(1-\alpha) > \sigma_\xi^2\left(1 + \frac{1}{\sigma_\eta^2}\right). \tag{A.17}$$

A.3 Equilibrium Information

I now solve for agent i's choice of information, taking as given aggregate information and the equilibrium mapping of information to actions. Following the derivation above, the average action from the perspective of agent i is given by the linear rule (A.1), where

$$\widetilde{\psi_1} = \left(n + \left(\frac{n}{\widetilde{k}} - \alpha\right)\sigma_\eta^2 \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)\right)^{-1} \tag{A.18}$$

$$\widetilde{\psi_2} = \left(1 + \sigma_\xi^2 \left(\frac{1}{1-\alpha} + \frac{\widetilde{k}}{1-\alpha\frac{\widetilde{k}}{n}}\frac{1}{\sigma_\eta^2}\right)\right)^{-1}. \tag{A.19}$$

Suppose that agent i selects to observe k signals and reacts to her information optimally according to the first order condition given by (2). Using the weights from agent i's action rule in (A.7)-(A.8), we can compute the differences

$$p^i - \theta = \left(k\psi_1^i + \psi_2^i - 1\right)\theta + \psi_1^i \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l + \psi_2^i \xi^i \tag{A.20}$$

$$\begin{aligned} p^i - \bar{p} = &\left(k\psi_1^i + \psi_2^i - n\widetilde{\psi_1} - \widetilde{\psi_2}\right)\theta + \left(\psi_1^i - \widetilde{\psi_1}\right)\sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l \\ &+ \widetilde{\psi_1}\sum_{l=1}^{n} \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\eta_l + \psi_2^i \xi^i. \end{aligned} \tag{A.21}$$

From here, compute the loss function and take expectations to get

$$\begin{aligned} -U^i\left(k^i, p^{i^*}(\mathcal{I}^i); \bar{p}(G)\right) = &(1-\alpha)\left(\left(k^i\psi_1^i + \psi_2^i - 1\right)^2 + {\psi_1^i}^2 k^i \sigma_\eta^2 + {\psi_2^i}^2 \sigma_\xi^2\right) \\ &+ \alpha\left(\left(k^i\psi_1^i + \psi_2^i - n\widetilde{\psi_1} - \widetilde{\psi_2}\right)^2 + \left(\psi_1^i - \widetilde{\psi_1}\right)^2 k^i \sigma_\eta^2 + \left(\widetilde{\psi_1}\right)^2 (n - k^i)\sigma_\eta^2 + {\psi_2^i}^2 \sigma_\xi^2\right) + \lambda k^i. \end{aligned} \tag{A.22}$$

A.3.1 Continuous Information

Fix exogenous parameters $\hat{\sigma}_\eta^2$ and $\hat{\lambda}$, which will correspond to the precision of the authority's communication and the cost of information in the continuous model. Now, consider a sequence of models indexed by parameter $\bar{n} \to \infty$, in which the public signal noise parameter is given by $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$ and the cost-per-signal is given by $\lambda = \frac{\hat{\lambda}}{\bar{n}}$. As $\bar{n}$ grows, the precision of a signal and its cost each become arbitrarily low.

The cost of information in each version of the model is invariant, in the sense that achieving a particular posterior variance on the state θ does not depend on $\bar{n}$. As an example, consider the cost of inferring the state with variance $\frac{\hat{\sigma}_\eta^2}{1+\hat{\sigma}_\eta^2}$ for a variety of $\bar{n}$. For any $\bar{n}$, doing so requires exactly $\bar{n}$ signals, since

$$E[(E(\theta|\{g_l; l = 1, ..., \bar{n}\}) - \theta)^2] = \frac{\sigma_\eta^2/\bar{n}}{1 + \sigma_\eta^2/\bar{n}} = \frac{\hat{\sigma}_\eta^2}{1 + \hat{\sigma}_\eta^2}. \tag{A.23}$$

The cost of observing $\bar{n}$ signals is always $\lambda\bar{n} = \hat{\lambda}$, establishing the invariance.[3]

[3]Expression (A.23) also establishes that, in terms of inference on the state, the information acquisition model here is identical to that of Myatt and Wallace (2012). That is, in terms of posterior variances of the state, it does not matter if I purchase $\bar{n}$ signals of variance $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$ or a single signal of variance $\hat{\sigma}_\eta^2$. The two models have very different implications for the cross-sectional correlation of information, however, which is crucial when agents interact strategically.

Agent i's loss function can be rewritten in terms of $\hat{\sigma}_\eta^2$ and the ratios $\frac{n}{\bar{n}}$ and $\frac{k^i}{\bar{n}}$. The limit of this function is well-defined, so long as the limits of these ratios are also well-defined. We can now define two parameters to summarize information choices, each of which can take on a continuous (rational) value. Let $\hat{n} = \lim_{\bar{n}\to\infty} \frac{n}{\bar{n}} \in [0, \infty)$, be the information authority's choice of scope. Next, let $\hat{k}^i = \lim_{\bar{n}\to\infty} \frac{k^i}{\bar{n}}$ be agent i's information choice. Since agents can only observe those signals released by the authority, $\hat{k}^i \in [0, \hat{n}]$. Finally, note that cost of information can be written $c(\hat{k}^i) = \hat{\lambda}\hat{k}^i$.

As $\bar{n}$ becomes large, the absolute value of ψ_1^i goes to zero. Let $\hat{\psi}_1^i = \lim_{\bar{n}\to\infty} \bar{n}\psi_1^i$ and $\hat{\psi}_2^i = \lim_{\bar{n}\to\infty} \psi_2^i$, so that

$$\hat{\psi}_1^i = \frac{1}{\hat{k}^i + \hat{\sigma}_\eta^2 + \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}} \left((1-\alpha) + \widetilde{\hat{\psi}_1} \left(\hat{n} + \hat{\sigma}_\eta^2 + \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2} \right) + \alpha \widetilde{\psi_2} \right) \tag{A.24}$$

$$\hat{\psi}_2^i = \frac{1}{\hat{k}^i \frac{\sigma_\xi^2}{\hat{\sigma}_\eta^2} + 1 + \sigma_\xi^2} \left((1-\alpha) + \alpha \widetilde{\hat{\psi}_1} (\hat{n} - \hat{k}^i) + \alpha \widetilde{\psi_2} \right). \tag{A.25}$$

Finally, taking the limit of expression (A.22), agent i's welfare can now be rewritten

$$-U^i\left(\hat{k}^i, p^{i^*}(\mathcal{I}^i); p(G)\right) = (1-\alpha)\left(\left(\hat{k}^i\hat{\psi}_1^i + \psi_2^i - 1\right)^2 + (\hat{\psi}_1^i)^2\hat{k}^i\hat{\sigma}_\eta^2 + (\hat{\psi}_2^i)^2\sigma_\xi^2 \right)$$
$$+ \alpha\left(\left(\hat{k}^i\hat{\psi}_1^i - \hat{n}\widetilde{\hat{\psi}_1}\right)^2 + (\hat{\psi}_1^i - \widetilde{\hat{\psi}_1})^2\hat{k}^i\hat{\sigma}_\eta^2 + \widetilde{\hat{\psi}_1}^2 (\hat{n} - \hat{k}^i)\sigma_\eta^2 + (\hat{\psi}_2^i - \widetilde{\psi_2})^2 + (\hat{\psi}_2^i)^2\sigma_\xi^2 \right) + \hat{\lambda}\hat{k}^i. \tag{A.26}$$

Aside from the substitution of variables with ˆ's, these equations are identical to their discrete counterparts in (A.18), (A.19) and (A.22). I suppress the distinction between $\hat{n}$ and n, etc, in the paper, but maintain it in the appendix for completeness.

A.3.2 Agent Loss is Convex

Twice-differentiating (A.26) with respect to $\hat{k}^i$ and simplifying substantially yields

$$-\frac{\partial^2 U^i}{\partial(\hat{k}^i)^2} = 2\frac{\hat{n}}{\hat{k}^i}\frac{\sigma_\xi^2\hat{\sigma}_\eta^2}{\widetilde{\hat{\psi}_1}} \frac{\left(\hat{\sigma}_\eta^2 + \sigma_\xi^2(\widetilde{\hat{k}}^i + \hat{\sigma}_\eta^2)\right)^2}{\left(\hat{\sigma}_\eta^2 + \sigma_\xi^2(\hat{k}^i + \hat{\sigma}_\eta^2)\right)^3} > 0. \tag{A.27}$$

So agent i's loss (utility) is convex (concave) on $\hat{k}^i \in [0, n]$.

A.3.3 Interior Levels of Acquisition

Agent i's problem is to find

$$\underset{\hat{k}^i}{\text{argmax}}\ U^i \qquad \text{subject to} \qquad 0 \leq \hat{k}^i \leq \hat{n}.$$

Let λ_1 and λ_2 be the multipliers on the inequality constraints $\hat{k}^i \leq \hat{n}$ and $\hat{k}^i \geq 0$ respectively. Then the agent's first order conditions are given by

$$0 = -\frac{\partial U^i}{\partial \hat{k}^i} + \lambda_1 - \lambda_2 + \lambda, \tag{A.28}$$

$$\lambda_1 \geq 0; \lambda_2 \geq 0, \tag{A.29}$$

and the complementary slackness conditions. A value of $\hat{k}^i$ that satisfies these conditions is a unique solution to the agent's optimization problem.

Differentiating agent welfare in equation (A.26) with respect to $\hat{k}^i$, and imposing equilibrium conditions $\hat{k}^i = \tilde{\hat{k}}$ yields the following expression:

$$-\hat{\sigma}_\eta^2 \left(\frac{\hat{n}}{\hat{k}} \hat{\psi}_1^* \right)^2 + \hat{\lambda} + \lambda_1 - \lambda_2 = 0. \tag{A.30}$$

For interior points, the extra Lagrange multipliers drop out to yield

$$\hat{\lambda} = \hat{\sigma}_\eta^2 \left(\frac{\hat{n}}{\hat{k}} \hat{\psi}_1^* \right)^2, \tag{A.31}$$

which can be solved for $\hat{k}$

$$\ddot{\hat{k}}(\hat{n}) = \frac{\left(\frac{\sigma_\eta^2}{\hat{\lambda}} \right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2 \tau}{1 - \frac{\alpha}{\hat{n}} \hat{\sigma}_\eta^2 \tau,} \tag{A.32}$$

where $\tau = \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2} \right)$.

A.3.4 Total Information Acquisition

Full information acquisition is an equilibrium if and only if individual i's loss is (weakly) decreasing in $\hat{k}^i$ at $\hat{k}^i = \hat{n}$, when other agent's information is also full ($\tilde{\hat{k}} = \hat{n}$.) When $\tilde{\hat{k}} = \hat{n}$, we have that $\hat{\psi}_1^* = \left(\hat{n} + \hat{\sigma}_\eta^2 + (1-\alpha) \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2} \right)^{-1}$ and the required inequality is

$$\lambda \leq \hat{\sigma}_\eta^2 \left(\hat{n} + \hat{\sigma}_\eta^2 + (1-\alpha) \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2} \right)^{-2}. \tag{A.33}$$

Rearrange the inequality to show that full acquisition is an equilibrium whenever

$$\hat{n} \leq \left(\frac{\hat{\sigma}_\eta^2}{\lambda} \right)^{\frac{1}{2}} - \sigma_\eta^2 - (1-\alpha) \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}. \tag{A.34}$$

A.3.5 No Information Acquisition

Conversely, no information acquisition is an equilibrium if and only if agent i's loss is (weakly) increasing in $\hat{k}^i$ at $\hat{k}^i = 0$, when other agents' information is also nil. Taking care to avoid dividing by zero, the agent's first order condition can be rearranged when $\widetilde{\hat{k}} = 0$ to yield the condition

$$\lambda \geq \frac{\hat{\sigma}_\eta^2}{\tau^2 \left(\hat{\sigma}_\eta^2 + \hat{k} \frac{\sigma_\xi^2}{1+\sigma_\xi^2} \right)^2}. \tag{A.35}$$

Evaluating at $\hat{k} = 0$, the condition for no information acquisition to be an equilibrium is

$$\lambda \geq \frac{1}{\sigma_\eta^2 \tau^2}. \tag{A.36}$$

Some parameter constellations satisfy both conditions (A.34) and (A.36). In this case, the model has two pure strategy equilibria. However, no interior value of $\hat{k}$ can simultaneously satisfy either the full information or no information conditions equation along with condition (A.31) for an interior equilbrium. By maintaining assumption 1, uniqueness is assured and the analysis is simplified.

B Social Welfare Function

In a symmetric equilibrium for a given $\hat{n}$, $\hat{\psi}_1^i = \frac{\hat{n}}{\hat{k}} \psi_1^*$ and $\hat{\psi}_2^i = \psi_2^*$. Evaluating equilibrium actions, social welfare can be written as

$$\begin{aligned} -U^G \left(\hat{k}, p^*(\mathcal{I}) \right) = (1 - \alpha^\star) \left(\left(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1 \right)^2 + \left(\hat{\psi}_1^* \hat{n} \right)^2 \frac{\hat{\sigma}_\eta^2}{\hat{k}} + \left(\hat{\psi}_2^* \right)^2 \sigma_\xi^2 \right) \\ + \alpha^\star \left((\hat{n}\hat{\psi}_1^*)^2 \left(1 - \frac{\hat{k}^*}{\hat{n}} \right)^2 \frac{\hat{\sigma}_\eta^2}{\hat{k}} + (\hat{n}\hat{\psi}_1^*)^2 \left(1 - \frac{\hat{k}}{\hat{n}} \right) \frac{\hat{\sigma}_\eta^2}{\hat{n}} + \left(\hat{\psi}_2^* \right)^2 \sigma_\xi^2 \right) + \hat{\lambda}\hat{k}. \end{aligned} \tag{B.1}$$

A great deal of simplification yields the following expression

$$-U^G = \hat{\sigma}_\eta^2 \left(\frac{n}{k} \hat{\psi}_1^* \right)^2 \left(\hat{k} \frac{1 - \frac{\hat{k}}{\hat{n}} \alpha^\star}{1 - \frac{\hat{k}}{\hat{n}} \alpha} + \left(1 - \frac{\hat{k}}{\hat{n}} \alpha \right) \hat{\sigma}_\eta^2 \left(\frac{1}{\sigma_\xi^2} + \frac{1 - \alpha^\star}{(1 - \alpha)^2} \right) \right) \left(1 - \frac{\hat{k}}{\hat{n}} \alpha \right) + \hat{\lambda}\hat{k}. \tag{B.2}$$

When $\alpha = \alpha^\star$, this is just

$$-U^G = \hat{\sigma}_\eta^2 \left(\frac{n}{k} \hat{\psi}_1^* \right) \left(1 - \frac{\hat{k}}{\hat{n}} \alpha \right) + \hat{\lambda}\hat{k}. \tag{B.3}$$

C Optimal Communication: Aligned Preferences

C.1 Proof of Lemma 1

Proof of Lemma 1. The form of $\hat{k}^*$ ensures that for any value $\hat{n}$ that implies $\hat{k}^*(\hat{n}) < \hat{n}$, there exists another value $\hat{n}' < \hat{n}$, such that $\hat{k}^*(\hat{n}') = \hat{n}' = \hat{k}^*(\hat{n})$. I want to show that social welfare is always higher under communication policy $\hat{n}'$. Using the simplified expression for social welfare derived in B.3, welfare under $\hat{n}'$ is greater if and only if

$$\frac{\hat{k} + (1-\alpha)\sigma_\eta^2\tau}{\hat{k} + \left(1 - \frac{\hat{k}^*}{\hat{n}}\alpha\right)\sigma_\eta^2\tau} \geq \frac{1-\alpha}{1 - \frac{\hat{k}^*}{\hat{n}}\alpha}. \tag{C.1}$$

But, since $\frac{\hat{k}^*}{n} < 1$ this must always be true. □

C.2 Optimal Scope

Using the result of lemma 1, I compute the optimal choice of $\hat{k}$ assuming that $\hat{k} = \hat{n}$. I then confirm that, under the implied policy, agents do choose $\hat{k}^* = \hat{n}$. If so, this represents the optimal level of transparency.

Under full acquisition, the social planner seeks to minimize loss given by

$$-U^G = (1-\alpha^\star)\left[(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1)^2 + \left(\hat{\psi}_1^*\right)^2 \hat{n}\hat{\sigma}_\eta^2\right] + \left(\hat{\psi}_2^*\right)^2 \sigma_\xi^2 + \lambda\hat{n}. \tag{C.2}$$

The first order condition is

$$-(1-\alpha^\star)\hat{\sigma}_\eta^2\left(\hat{\psi}_1^*\right)^2 + 2\left(\hat{\psi}_1^*\right)^3\left(\hat{\sigma}_\eta^2\right)^2\frac{1-\alpha}{\sigma_\xi^2}(\alpha - \alpha^\star) + \lambda = 0. \tag{C.3}$$

When preferences are aligned, this reduces to

$$(1-\alpha)\hat{\sigma}_\eta^2\left(\hat{\psi}_1^*\right)^2 = \lambda, \tag{C.4}$$

which can be solved for $\hat{n}^\star$:

$$\hat{n}^\star = \sqrt{\frac{(1-\alpha)\hat{\sigma}_\eta^2}{\lambda}} - \hat{\sigma}_\eta^2 - (1-\alpha)\frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}. \tag{C.5}$$

Inspection shows that this is less than the threshold value $\widehat{\hat{n}}$, and so the result is established.

D Constraints on Authority's own Information

One question that arises frequently in the literature on transparency is whether the degree of the authority's knowledge about the state should impact how it communicates its message. The canonical model with a single public signal does not distinguish between limitations on the authority's knowledge of the state and limitations on its ability to clearly communicate that knowledge. In this section, I extend the model to allow for error in the authority's own knowledge of the state. The key characteristic of this type of error is that it is common across the authority's signals: every time it "speaks," the authority makes the same mistake because it misapprehends the realization of the state.

To this end, assume the authority learns about the state from a signal of the form $g = \theta + \varepsilon$. The authority, in turn, may freely release as many signals $g_l = \theta + \varepsilon + \eta_l, l = 1, ..., n$, as it wishes. The error term ε is assumed to be independent of all other shocks and normally distributed with variance σ_ε^2. All other assumptions in the model are unchanged.

D.1 Equilibrium Actions

Suppose $\sigma_\varepsilon^2 > 0$. Then, the conditional expectation of signal l is given by

$$E\left(g_l|\mathcal{I}^i\right) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta + \varepsilon|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{D.1}$$

Agent i's conditional expectation of the state and of the authority's "mistake" are now given respectively by

$$E(\theta|\mathcal{I}^i) = \gamma_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + \gamma_2 r^i \tag{D.2}$$

$$E(\varepsilon|\mathcal{I}^i) = a_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + a_2 r^i, \tag{D.3}$$

where $\gamma_1 = \frac{\sigma_\xi^2}{\chi + \sigma_\xi^2(k+\chi)}$, $\gamma_2 = \frac{\chi}{\chi + \sigma_\xi^2(k+\chi)}$, $a_1 = \frac{\sigma_\varepsilon^2\left(1+\sigma_\xi^2\right)}{\chi + \sigma_\xi^2(k+\chi)}$, $a_2 = -\frac{k\sigma_\varepsilon^2}{\chi + \sigma_\xi^2(k+\chi)}$, and $\chi = k\sigma_\varepsilon^2 + \sigma_\eta^2$.

Individual i's action is given by

$$\begin{aligned} p^i =& \left[(1-\alpha)\gamma_1 + \alpha\left(\psi_1(1 + (n-k)(\gamma_1 + a_1)) + \psi_2\gamma_1\right)\right] \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \\ &+ \left[(1-\alpha)\gamma_2 + \alpha\left(\psi_1(n-k)(\gamma_2 + a_2) + \psi_2\gamma_2\right)\right] r^i. \end{aligned} \tag{D.4}$$

Once again computing the expectation of the aggregate action gives

$$E(\bar{p}|\mathcal{I}^i) = (\psi_1(1+(n-k)(\gamma_1+a_1)) + \psi_2\gamma_1)\sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \\ + (\psi_1(n-k)(\gamma_2+a_2) + \psi_2\gamma_2)\, r^i. \tag{D.5}$$

Finding the fixed point as before yields the equilibrium coefficients,

$$\psi_1^* = \left(n\Gamma + \left(\frac{n}{k} - \alpha\right)\sigma_\eta^2\left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)\right)^{-1} \tag{D.6}$$

$$\psi_2^* = \left(\Gamma + \sigma_\xi^2\left(\frac{1}{1-\alpha} + \frac{k}{(1-\alpha)k\sigma_\varepsilon^2 + \left(1-\alpha\frac{k}{n}\right)\frac{1}{\sigma_\eta^2}}\right)\right)^{-1}, \tag{D.7}$$

where $\Gamma \equiv 1 + \sigma_\varepsilon^2 + (1-\alpha)\frac{\sigma_\varepsilon^2}{\sigma_\xi^2}$.

D.2 Equilibrium Information

The agent's problem is exactly as in A.3.3 and the first order conditions are identical (up to the relevant definition of $\hat{\psi}_1^*$.)

$$-\hat{\sigma}_\eta^2\left(\frac{n}{k}\hat{\psi}_1^*\right)^2 + \lambda + \lambda_1 - \lambda_2 = 0. \tag{D.8}$$

The derivations of equilibrium information follow exactly as before. One potentially surprising result is that the condition for zero information to be an equilibrium does not change. That is, there is no need to restate assumption 1 as long $\sigma_\varepsilon^2 < \infty$.

Proposition 2(c) describes the equilibrium information choice of agents

Proposition 2(c). *Suppose that assumption 1 holds and that σ_ε^2 is finite. Then the equilibrium information allocation is unique and is given by*

$$k^* = \begin{cases} n & \text{if } n \le \left(\sqrt{\frac{\sigma_\eta^2}{\lambda}} - (1-\alpha)\sigma_\eta^2\tau\right)\frac{1}{\Gamma} \\ \ddot{k}(n) & \text{otherwise} \end{cases} \tag{D.9}$$

where

$$\ddot{k}(n) = \frac{\sqrt{\frac{\sigma_\eta^2}{\lambda}} - \sigma_\eta^2\tau}{\Gamma - \frac{\alpha}{n}\sigma_\eta^2\tau}. \tag{D.10}$$

Since $\Gamma > 1$, the threshold at which agents cease to observe all signals released by the authority necessarily shrinks. Furthermore, to the right of the threshold, the derivative

$$\frac{\partial \ddot{k}(n)}{\partial \sigma_\varepsilon^2} = -\ddot{k}(n) \frac{\sigma_\varepsilon^2 \left(1 + \frac{1-\alpha}{\sigma_\xi^2}\right)}{\Gamma - \frac{\alpha}{n}\sigma_\eta^2 \tau} < 0. \tag{D.11}$$

Thus, agent's acquisition of public signals is always less when the authority knows less about the state.

D.3 Choice of Scope

Social welfare under the assumption $\hat{k} = \hat{n}$ is now written

$$U^G = (1-\alpha)\left[\left(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1\right)^2 + \left(\hat{\psi}_1^*\right)^2 \hat{n}\hat{\sigma}_\eta^2 + \left(\hat{n}\hat{\psi}_1^*\right)^2 \sigma_\varepsilon^2\right] + \left(\hat{\psi}_2^*\right)^2 \sigma_\xi^2 + \lambda \hat{n}. \tag{D.12}$$

Taking the first order condition and solving for $\hat{n}^\star$ yields

$$\hat{n}^\star = \left(\sqrt{\frac{\hat{\sigma}_\eta^2(1-\alpha)}{\hat{\lambda}}} - (1-\alpha)\hat{\sigma}_\eta^2 \tau\right)\frac{1}{\Gamma}. \tag{D.13}$$

Proposition 4(c) summarizes optimal scope in this case.

Proposition 4(c). *Suppose that assumption 1 holds and that σ_ε^2 is finite. Then the optimal choice of scope is given by*

$$n^\star = \left(\sqrt{\frac{\sigma_\eta^2(1-\alpha)}{\lambda}} - (1-\alpha)\sigma_\eta^2 \tau\right)\frac{1}{\Gamma}. \tag{D.14}$$

The proposition shows that optimal scope with $\sigma_\varepsilon^2 > 0$ is just a rescaling of the optimal level when the authority's information is perfect. Proposition 4(c) confirms an intuitive result: an authority which knows less about the state should provide fewer public signals about it.

Since errors in the authority's knowledge of the state are common across all public signals, they bear on the informativeness of the public signals with regard to the fundamental, but are not directly related to agents' coordination problem. For this reason, it is natural to suppose that increasing the magnitude of such errors is always harmful to social welfare. Proposition 5(c) establishes this result for the case of aligned preferences.

Proposition 5(c). *Given n, equilibrium social welfare is increasing in the precision of the authority's own information. That is,*

$$\frac{\partial U^G}{\partial \sigma_\varepsilon^2} < 0. \tag{D.15}$$

Proof. Again, by proposition 3 in the main text, it is sufficient to show this holds for a given choice of k. The proposition then follows directly from the fact that actions given information are efficient when preferences are aligned. □

In summary, the addition of uncertainty in the authority's own apprehension of the state confirms some natural conjectures, but it does not affect the basic mechanism of the model. The results on information acquisition and optimal scope follow with minimal modification.

E Optimal Communication: Misaligned Preferences

E.1 Private Information

Proof of Proposition 4(b). Suppose $\hat{\sigma}_\eta^2$ is given. Then, when $\alpha \leq \alpha^\star$, the authority always prefers that agents acquire full information. To see this compare social welfare for a given $\hat{k}$, for the case that $\hat{k} < \hat{n}$ and the case that $\hat{k} = \hat{n}$. In many cases, full acquisition may also be optimal when $\alpha > \alpha^\star$ as well. In these cases, it follows directly that $\hat{n}^\star$ is finite.

Now suppose $\alpha > \alpha^\star$ and the the authority finds it optimal to select $\hat{n}^o > \widehat{\hat{n}}$. To find $\hat{n}^o$ in this case, plug in agents' information acquisition $\ddot{k}(\hat{n})$, take the first order condition with respect to $\hat{n}$, and solve. This first order condition has a unique (real) solution, $\hat{n}^o$, and it is finite. To show that this is indeed an optimum, however, requires slightly more work. Taking the second order condition and evaluating at $\hat{n}^o$ shows that the loss function is locally convex, and since there is only one critical point, that this is indeed the global optimum.

The general expression for $\hat{n}^o$ is unwieldy. However, when $\alpha^\star = 0$ it simplifies substantially to

$$\hat{n}^o = (1-\alpha)^2 \left(\sqrt{\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}} \left(\frac{1}{(1-\alpha)^2} + \frac{1}{\sigma_\xi^2} \right) - \hat{\sigma}_\eta^2 \tau^2 \right). \tag{E.1}$$

□

Proof of Proposition 5(b). It suffices to show that welfare is improving in $\frac{1}{\hat{\sigma}_\eta^2}$, given optimal scope, for any level $\hat{\sigma}_\eta^2$. If optimal scope calls for full acquisition, the authority can always decrease scope to maintain the same total precision and benefit from lower information costs. I now demonstrate that when optimal scope lies to the right of $\widehat{\hat{n}}$, welfare is again improving if scope is adjusted optimally. Substitute in $\ddot{k}(\hat{n})$, and then $\hat{n}^o$, into the social welfare function given in (B.2). The derivative with respect to $\hat{\sigma}_\eta^2$ is greater than zero by assumption 1, and the result is established. □

F Directed Search

In this section, I solve a version of the model with a more general type of information choice, in which agents may choose the probability with which they observe particular signals, but at a cost. The logic parallels that of section A.1, despite some added complications.

Assume now that agents assign relative weights $w_1, w_2, ..., w_n$ to each signal released by the information authority, so that the probability of drawing the j'th signal as the first signal drawn is $\frac{w_j}{\sum_{l=1}^{n} w_l}$. I again assume that signals are drawn sequentially, without replacement. Because signals are drawn without replacement, the probability that g_j is drawn on the second draw depends on which signal was drawn in the first round, and so on. The distribution characterizing this search process is know as the generalized *Wallenius noncentral hypergeometric distribution.*

Let $\mu_l = P(\mathcal{G}_l^i = 1; ||\mathcal{G}^i|| = k)$ be the probability that signal l is drawn among a sample of k signals. Unfortunately, for $k > 1$ there is no requirement that μ_l is proportional to w_l and, in fact, there is no closed-form solution for μ_l as a function of the w_l's. Chen et al. (1994) show, however, that a set of w_l's can be mapped unquietly into a set of μ_l's and the two respect a natural ordering relation

$$w_l > w_j \iff \mu_l > \mu_j. \tag{F.1}$$

To simplify the analysis, and because the agents care directly about μ_l, I proceed as if these are the fundamental choice of the agents, although they could always be mapped back into a set of weights used for the sampling process. The μ_l's also have the important property that $\sum_{l=1}^{n} \mu_l = k$.

F.1 Equilibrium Actions

The equilibrium pricing rule must reflect the fact that some signals may, in general, be observed by more agents than others. Therefore I guess the following form for a linear equilibrium

$$\bar{p} = \sum_{l=1}^{n} \tilde{\psi}_l g_l. \tag{F.2}$$

Under the baseline information assumptions, we have that

$$E(\theta|\mathcal{I}^i) = \frac{1}{k + \sigma_\eta^2} \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \tag{F.3}$$

$$E(g_l|\mathcal{I}^i) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{F.4}$$

Optimal action on the part of agent i implies

$$
\begin{aligned}
p^i &= \frac{(1-\alpha)}{k+\sigma_\eta^2}\sum_{l=1}^{n}\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l + \alpha\sum_{l=1}^{n}\tilde{\psi}_l\left[\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l + \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\frac{1}{k+\sigma_\eta^2}\sum_{j=1}^{n}\mathbb{1}\left[g_j \in \mathcal{I}^i\right]g_j\right] \\
&= \sum_{l=1}^{n}\left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\tilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2}\sum_{j=1}^{n}\tilde{\psi}_j\mathbb{1}\left[g_j \notin \mathcal{I}^i\right]\right]\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l \\
&\equiv \sum_{l=1}^{n}\hat{\psi}_l^i\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l. \qquad \text{(F.5)}
\end{aligned}
$$

where $\hat{\psi}_l^i$ is agent i's optimal response to signal l conditional on $g_l \in \mathcal{I}^i$.

At this point, a new complication arises in that $\hat{\psi}_l^i$ is a random variable, both cross sectionally and from the perspective of agent i. This randomness is problematic because $\hat{\psi}_l^i$ and $\mathbb{1}[g_l \in \mathcal{I}^i]$ are not independent and no closed form expression exists for their covariance. This complicates the step of integrating across agents in order to determine the aggregate action rule. The key observation required to circumvent this difficulty is that, as $\bar{n}$ grows larger, $\hat{\psi}_l^i$ becomes essentially deterministic. This allows for both straightforward aggregation across agents and simple computation of expected values.

To make this claim more precise, consider once again a sequence of models indexed by $\bar{n}$, in which $\lim_{\bar{n}\to\infty}\frac{n}{\bar{n}} = \hat{n}$ and $\lim_{\bar{n}\to\infty}\frac{k}{\bar{n}} = \hat{k}$, and $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$. Define the set of random variables $x_j = n\widetilde{\psi}_{\widetilde{l}_j}\mathbb{1}\left[g_{\widetilde{l}_j} \notin \mathcal{I}^i\right]$, where the indexes $\widetilde{l}_j, j = 1, 2, ..., n$, are generated by randomly drawing an index l, without replacement, from among the n public signals. Define

$$
b_{\bar{n}} \equiv E[x_1] = \frac{1}{n}\sum_{l=1}^{n}n\widetilde{\psi}_l(1-\mu_l) \qquad \text{(F.6)}
$$

$$
\delta_{\bar{n}} \equiv \frac{1}{n}\sum_{j=1}^{n}n\widetilde{\psi}_l\mathbb{1}\left[g_j \notin \mathcal{I}^i\right] - b_{\bar{n}} = \frac{1}{n}\sum_{j=1}^{n}x_j - b_{\bar{n}}. \qquad \text{(F.7)}
$$

Then, equation (F.5) can be rewritten after some manipulation as

$$
p^i = \sum_{l=1}^{n}\left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2}\left(b_{\bar{n}} + \delta_{\bar{n}}\right)\right]\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l. \qquad \text{(F.8)}
$$

Integrate across agents to get

$$
\bar{p} = \sum_{l=1}^{n}\left(\frac{\alpha}{k+\sigma_\eta^2}\Delta_{\bar{n},l} + \mu_l\left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2}b_{\bar{n}}\right]\right)g_l, \qquad \text{(F.9)}
$$

where $\Delta_{\bar{n},l} \equiv E\left[\delta_{\bar{n}} \mathbb{1}[g_l \in \mathcal{I}^i]\right]$. The equilibrium coefficients are then given by the fixed point of the expression

$$\widetilde{\psi}_l = \frac{\alpha}{k+\sigma_\eta^2}\Delta_{\bar{n},l} + \mu_l\left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2}b_{\bar{n}}\right]. \tag{F.10}$$

Now, solving for $\widetilde{\psi}_l$ yields

$$\widetilde{\psi}_l = \frac{\mu_l}{1-\alpha\mu_l}\frac{1}{k+\sigma_\eta^2}\left(1-\alpha+\alpha b_{\bar{n}}\right) + \frac{\alpha}{(1-\alpha\mu_l)(k+\sigma_\eta^2)}\Delta_{\bar{n},l}. \tag{F.11}$$

Now, using the fact that $-E(|\delta_{\bar{n}}|) \leq \Delta_{\bar{n},l} \leq E(|\delta_{\bar{n}}|)$, we have the inequality

$$\widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l}\frac{1}{k+\sigma_\eta^2}\left(1-\alpha+\alpha b_{\bar{n}}\right) + \frac{\alpha}{(1-\alpha\mu_l)(k+\sigma_\eta^2)}E(|\delta_{\bar{n}}|) \tag{F.12}$$

and a corresponding lower bound on $\widetilde{\psi}_l$. Substituting recursively and simplifying yields the following bounds on $\widetilde{\psi}_l$

$$\frac{\mu_l}{1-\alpha\mu_l}\rho_1 - E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\rho_{2,l} \leq \widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l}\rho_1 + E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\rho_{2,l}, \tag{F.13}$$

where

$$\begin{aligned}
\rho_1 &= \frac{1-\alpha}{k+\sigma_\eta^2-\alpha q} \\
\rho_{2,k} &= \left(\frac{1}{k+\sigma_\eta^2} + \frac{\alpha\mu_l\bar{q}}{(k+\sigma_\eta^2)(k+\sigma_\eta^2-\alpha q)}\right) \\
q &= \sum_{l=1}^{n}\frac{(1-\mu_l)\mu_l}{1-\alpha\mu_l} \\
\bar{q} &= \sum_{l=1}^{n}\frac{(1-\mu_l)}{1-\alpha\mu_l}.
\end{aligned}$$

Now, multiply the inequality by $\bar{n}$, to get

$$\frac{\mu_l}{1-\alpha\mu_l}\bar{n}\rho_1 - E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\bar{n}\rho_{2,k} \leq \bar{n}\widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l}\bar{n}\rho_1 + E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\bar{n}\rho_{2,k}. \tag{F.14}$$

A law of large numbers applies to $\frac{1}{n}\sum_{j=1}^{n} x_j$, implying that $\lim_{\bar{n}\to\infty} E(|\delta_{\bar{n}}|) = 0$.[4] Let

$$Q \equiv \lim_{\bar{n}\to\infty}\frac{1}{\bar{n}}\sum_{l=1}^{n}\frac{(1-\mu_l)\mu_l}{1-\alpha\mu_l}. \tag{F.15}$$

[4] This follows from the construction of x_j as a sequence of *exchangeable* random variables. See McCall (1991) for a detailed discussion and additional references on the topic of exchangeability.

This is clearly finite, since each term in the summand is positive and bounded by a finite constant, while the total is divided by n. For the same reasons, $\bar{Q} \equiv \lim_{\bar{n}\to\infty} \frac{1}{\bar{n}} \sum_{l=1}^{n} \frac{1-\mu_l}{1-\alpha\mu_l}$ is also finite. Therefore, $\bar{n}\rho_1$ and $\bar{n}\rho_{2,l}$ each converge to a finite values and we can conclude that

$$\lim_{\bar{n}\to\infty} \bar{n}\widetilde{\psi}_l = \frac{\mu_l}{1-\alpha\mu_l}\rho \qquad \equiv \widetilde{\varphi}(l) \tag{F.16}$$

$$\lim_{\bar{n}\to\infty} \bar{n}\hat{\psi}_l = \frac{1-\alpha}{\hat{k}+\hat{\sigma}_\eta^2} + \alpha\widetilde{\varphi}_l + \frac{\alpha}{\hat{k}+\hat{\sigma}_\eta^2}\sum_{k=0}^{\infty}(1-\mu_l)\widetilde{\varphi}_l \qquad \equiv \varphi(l), \tag{F.17}$$

where $\rho = \frac{1-\alpha}{k+\hat{\sigma}_\eta^2-\alpha Q}$.

F.2 Agents' Information Choice

I now follow a similar strategy to compute the loss of agent i, taking aggregate actions as given. To begin, compute the deviations

$$p^i - \theta = \left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)\theta + \sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l \tag{F.18}$$

$$p^i - \bar{p} = \left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - \widetilde{\psi}_l\right)\theta + \sum_{l=1}^{n} \left(\hat{\psi}_l - \widetilde{\psi}_l\right)\mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l + \sum_{l=1}^{n} \widetilde{\psi}_l \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\eta_l \tag{F.19}$$

of the discrete model. We are interested in computing $E\left[(p^i-\theta)^2\right]$ and $E\left[(p^i-\bar{p})^2\right]$.

First, consider the "fundamental deviation" given by

$$\begin{aligned} E\left[(p^i-\theta)^2\right] &= E\left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)^2 + \sum_{l=1}^{n} E(\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \sigma_\eta^2 \\ &= E\left(\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)^2 + \frac{1}{\bar{n}}\sum_{l=1}^{n} E(\bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \frac{\sigma_\eta^2}{\bar{n}} \\ &= E\left[\left(\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\right)^2 - 2\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] + 1\right] \\ &\quad + \frac{1}{\bar{n}}\sum_{l=1}^{n} E(\bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \frac{\sigma_\eta^2}{\bar{n}}. \end{aligned} \tag{F.20}$$

Taking the limit $\bar{n} \to \infty$ and rewriting the infinite sum as an integral over the domain $l \in [0, \hat{n}]$, expression (F.20) now simplifies considerably to

$$E\left[(p^i-\theta)^2\right] = \left(\int_o^{\hat{n}} \mu(l)\varphi(l)dl - 1\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\varphi(l)^2 dl. \tag{F.21}$$

The limiting "coordination loss" term can be derived in the same manner:

$$E\left[(p^i-\bar{p})^2\right] = \left(\int_0^{\hat{n}} \mu(l)\left(\varphi(l)-\widetilde{\varphi}(l)\right)dl\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\left(\varphi(l)-\widetilde{\varphi}(l)\right)^2 dl + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} (1-\mu(l))\widetilde{\varphi}(l)^2 dl. \tag{F.22}$$

Finally, write the cost of information as the functional mapping $\mu(l)$ to the cost $c(\mu(l))$. Now, combining all terms yields agent i's welfare function

$$\begin{aligned}
-U^i =&(1-\alpha)\left[\left(\int_o^{\hat{n}} \mu(l)\varphi(l)dl - 1\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\varphi(l)^2 dl\right] \\
&+ \alpha\left[\left(\int_0^{\hat{n}} \left(\mu(l)\varphi(l)-\widetilde{\varphi}(l)\right)dl\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\left(\varphi(l)-\widetilde{\varphi}(l)\right)^2 dl\right. \\
&\left.+\hat{\sigma}_\eta^2 \int_0^{\hat{n}} (1-\mu(l))\widetilde{\varphi}(l)^2 dl\right] + c(\mu(l)).
\end{aligned} \tag{F.23}$$

The solution to agent i's problem is characterized by

$$\underset{\mu(l),\hat{k}}{\operatorname{argmax}}\ U^i \quad \text{subject to} \quad \mu(l)\le 1; \mu(l)\ge 0; \int_0^{\hat{n}} \mu(l)dl \le \hat{k}.$$

Let $\lambda_1(l), \lambda_2(l), \lambda_3$ be the Lagrange multipliers on the three constraints respectively.

F.2.1 Equilibrium Characterization

Taking derivatives with respect to $\mu(l)$ and $\hat{k}$ yields

$$\begin{aligned}
0 = &2(1-\alpha)\left[(\phi-1)\left(\varphi(l)-\frac{\alpha\hat{k}}{\hat{k}+\hat{\sigma}_\eta^2}\widetilde{\varphi}(l)\right) + \hat{\sigma}_\eta^2\left(\frac{\varphi^2(l)}{2} - \frac{\alpha\widetilde{\varphi}(l)}{\hat{k}+\hat{\sigma}_\eta^2}\int_0^{\hat{n}} \mu(j)\varphi(j)dj\right)\right] \\
&+ 2\alpha\left[\left(\phi - \int_0^{\hat{n}} \widetilde{\varphi}(j)dj\right)\left(\varphi(l)-\frac{\alpha\hat{k}}{\hat{k}+\hat{\sigma}_\eta^2}\widetilde{\varphi}(l)\right) + \hat{\sigma}_\eta^2\left(\frac{\varphi(l)^2}{2} - \varphi(l)\widetilde{\varphi}(l)\right.\right. \\
&\left.\left. -\ \widetilde{\varphi}(l)\frac{\alpha}{\hat{k}+\hat{\sigma}_\eta^2}\int_0^{\hat{n}} \mu(j)(\varphi(j)-\widetilde{\varphi}(j))dj\right)\right] + \lambda_1(l) - \lambda_2(l) + \lambda_3 + c_l(\mu(l))
\end{aligned} \tag{F.24}$$

$$\begin{aligned}
-\lambda_3 = &2(1-\alpha)\frac{\partial\varphi}{\partial\hat{k}}\left[(\phi-1)\int_0^{\hat{n}} \mu(j)dj + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(j)\varphi(j)dj\right] \\
&+ 2\alpha\frac{\partial\varphi}{\partial\hat{k}}\left[\left(\phi - \int_0^{\hat{n}} \widetilde{\varphi}(j)dj\right)\int_0^{\hat{n}} \mu(j)dj + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(j)\left(\varphi(j)-\widetilde{\varphi}(j)\right)dj\right],
\end{aligned} \tag{F.25}$$

where $\phi = \int_0^{\hat{n}} \mu(l)\varphi(l)dl$ and

$$\frac{\partial\varphi}{\partial\hat{k}} \equiv -\left(\frac{1}{\hat{k}+\hat{\sigma}_\eta^2}\right)^2\left((1-\alpha)+\alpha\int_0^{\hat{n}}(1-\mu(j))\widetilde{\varphi}(j)dj\right) = \frac{\partial\varphi(l)}{\partial\hat{k}} \tag{F.26}$$

is constant across l.

Substituting the equilibrium relationships

$$\hat{k} = \int_0^{\hat{n}} \mu(j)dj \tag{F.27}$$

$$\widetilde{\varphi}(l) = \mu(l)\varphi \tag{F.28}$$

$$\varphi = \left(\frac{1}{1-\alpha\mu(l)}\right)\rho \tag{F.29}$$

into equation (F.24) and simplifying substantially yields

$$2(1-\alpha)\rho(\phi-1)+\hat{\sigma}_\eta^2\rho^2\frac{2\alpha^2\mu(l)^2-4\alpha\mu(l)+1}{(1-\alpha\mu(l))^2}+\lambda_1(l)-\lambda_2(l)+\lambda_3+c_l(\mu(l))=0.$$

Additional algebra shows that $(\phi-1) = -\frac{1}{\hat{k}+\hat{\sigma}_\eta^2-\alpha Q}\hat{\sigma}_\eta^2$. Using this result, the first order condition simplifies further to

$$-\hat{\sigma}_\eta^2\left(\frac{\rho}{1-\alpha\mu(l)}\right)^2+\lambda_1(l)-\lambda_2(l)+\lambda_3+c_l(\mu(l))=0. \tag{F.30}$$

Furthermore, algebraic manipulations of (F.25) establishes that in equilibrium $\lambda_3 = 0$.

Let $\bar{\lambda}_1(l) = \lambda_1(l)\left(1-\alpha\mu(l)\right)^2$ and $\bar{\lambda}_2(l) = \lambda_2(l)\left(1-\alpha\mu(l)\right)^2$. Proposition 7 combines the above results to characterize the set of equilibria in the extended model.

Proposition 7. *The set of equilibria in the model are characterized by the set of equalities indexed by l,*

$$c_l(\mu(l))\left(1-\alpha\mu(l)\right)^2 = \hat{\sigma}_\eta^2\rho^2-\bar{\lambda}_1(l)+\bar{\lambda}_2(l), \tag{F.31}$$

the inequality constraints

$$\mu(l)\leq 1; \mu(l)\geq 0; \int_0^{\hat{n}}\mu(l)dl\leq\hat{k}, \tag{F.32}$$

the complementarity slackness conditions, $\bar{\lambda}_1(l)(\mu(l)-1)=0$*,* $\bar{\lambda}_2(l)\mu(l)=0$*, and the inequalities,* $\bar{\lambda}_1(l)\geq 0, \bar{\lambda}_2(l)\geq 0$.

F.3 A Sufficient Condition for Uniqueness

Suppose that the cost of information is given by the CES aggregator in equation (29), with $\omega > 1$. The derivative of cost with respect to $\mu(l)$ is

$$c_l(\mu(l)) = \lambda \hat{n}^{\frac{\omega-1}{\omega}} \left(\int_0^{\hat{n}} \mu(l)^\omega dl \right)^{\frac{1-\omega}{\omega}} \mu(l)^{\omega-1}. \tag{F.33}$$

The model has a unique equilibrium whenever the left-hand side of (F.31) is monotonically increasing in $\mu(l)$. To see this, note first that one can immediately rule out $\mu(l) = 0$, since the derivative of the cost function with respect to $\mu(l)$ is always zero when $\mu(l) = 0$. Second, note that if $\mu(l) \in (0, 1)$ satisfies

$$c_l(\mu(l)) \left(1 - \alpha\mu_l\right)^2 = \hat{\sigma}_\eta^2 \rho^2 \tag{F.34}$$

for any k, then monotonicity implies that $c_l(\mu(l)) \left(1 - \alpha\mu_l\right)^2 > \hat{\sigma}_\eta^2 \rho^2$ at $\mu(l) = 1$, ruling out the possibility that $\lambda_1(l) \geq 0$, and therefore that $\mu(l) = 1$, for any k. Finally, when $\bar{\lambda}_1(l) = \bar{\lambda}_2(l) = 0$ and the lefthand side is monotonic, only one value $\mu(l)$ can simultaneously satisfy equation (F.31), so that $\mu(l) = \nu$ and the equilibrium conditions reduce to those from the baseline model.

The required monotonicity is achieved whenever

$$\mu(l)^{\omega-1}(1 - \alpha\mu(l))^2 \tag{F.35}$$

is monotonic on $[0, 1]$. Taking a derivative and imposing the inequality quickly establishes the requirement that

$$\omega > \frac{1+\alpha}{1-\alpha}. \tag{F.36}$$

F.4 Multiple Equilibria when Information Cost is Linear

Suppose now that the derivative of the cost function $c_l(\mu(l)) = \hat{\lambda}$. An immediate implication of proposition 7 is that, in equilibrium, the function $\mu(l)$ can takes on no more than one interior value, in addition to $\mu(l) = 0$ or $\mu(l) = 1$. To see this, consider expression (F.31) for a value of l for which neither constraint one nor constraint two is binding. In this case, the left-hand side of equation (F.31) is strictly decreasing in $\mu(l)$, implying that no more than one interior value of $\mu(l)$ can simultaneously satisfy the equation. In contrast to the case where the left hand side is increasing, however, it still may be that $\bar{\lambda}_1(l) > 0$ or $\bar{\lambda}_2(l) > 0$ or both, creating the potential for a great deal of multiplicity.

Imposing the restriction that $\mu(l)$ takes on no more than one interior value, a set of simple conditions can be derived characterizing the set of equilibria in the model.

Let $\hat{n}_1, \hat{n}_2, \hat{n}_3; \hat{n} \geq \hat{n}_i \geq 0; \hat{n} = \sum_{i=1}^{3} \hat{n}_i$ denote the "mass" of signals taking on values $\mu^* \in (0,1), \bar{\mu} = 1, \underline{\mu} = 0$, respectively. Solving the first order condition for μ^* yields

$$\mu^* = \frac{(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2 - \hat{n}_2}{(1-\alpha)\hat{n}_1 - \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)}. \tag{F.37}$$

Since $\{\hat{n}_1, \hat{n}_2\}$ imply values for μ^* and $\hat{n}_3$, they are sufficient to characterize all equilibria. Assumption 1 ensures that agents acquire at least some information, so that $\hat{n}_1 + \hat{n}_2 > 0$. Furthermore, if $\hat{n}_1 + \hat{n}_2 < \hat{n}$, the requirement that $\lambda_1(l) \geq 0$ and $\lambda_2(l) \geq 0$ ensures $\mu^* \in [0,1]$. Proposition 8 describes the necessary and sufficient conditions for this.

Proposition 8. *Suppose that the cost of information is given by $c(\mu(l)) = \lambda\hat{k}$. The the set of equilibria is characterized by $\{\hat{n}_1, \hat{n}_2\}$ that satisfy one of the two sets of conditions below*

- *Case 1: Full Acquisition Only:* $\hat{n} \leq (1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2$

    1. $\hat{n}_1 = 0$ *and* $\hat{n}_2 = \hat{n}$

- *Case 2: Multiple Equilibria:* $\hat{n} > (1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2$

    - *Case 2a*
        1. $(1-\alpha)\hat{n}_1 < \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)$
        2. $\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \geq \hat{\sigma}_\eta^2 + \hat{n}_1 + \hat{n}_2$
        3. $(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \leq \hat{\sigma}_\eta^2 + \hat{n}_2$
    - *Case 2b*
        1. $(1-\alpha)\hat{n}_1 > \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)$
        2. $\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \leq \hat{\sigma}_\eta^2 + \hat{n}_1 + \hat{n}_2$
        3. $(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \geq \hat{\sigma}_\eta^2 + \hat{n}_2$

The two-by-two panel in figure 1 captures the range of multiple equilibria for different degrees of strategic complementarity, at different levels of scope. In the first row, with relatively low scope, equilibrium always entails some degree of perfectly directed search by agents. This result is closely related to the result in the baseline model where, for low levels of scope, agents always choose to observe all signals. In this case, if agent i can be assured that no others observe a particular signal k,

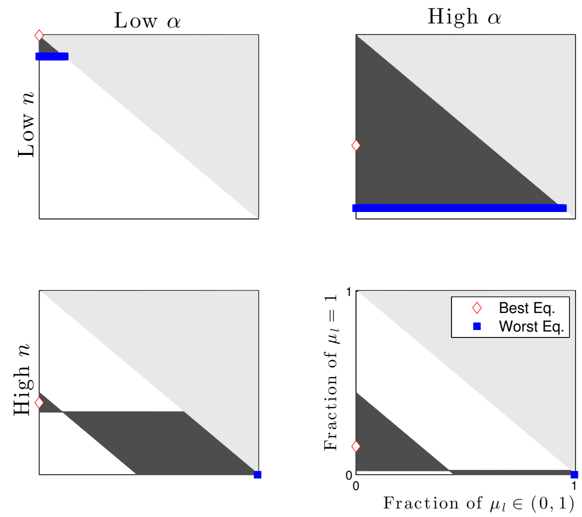

Figure 1: Multiple equilibria for different degrees of scope and strategic complementarity. Dark-shaded region denotes equilibrium points, white region denotes points which are not equilibria. Light-shaded (upper-right) region is not feasible. The "best" equilibria never entail randomization.

then she may choose not to observe it as well. However, if others do observe that signal with positive probability, then she desires to do so as well, which increases the signal's informativeness, causing others to increase the probability with which they draw that signal, and so on. Once agents have acquired enough signals in this directed manner, however, this logic no longer bites, and agents may choose undirected search over some of the remaining signals.

Higher strategic complementarities generally increase the scope for multiplicity. In the current model, this effect is apparent when scope is low. When scope is high, however, the consequences for multiplicity are more subtle. This contrast stems from the fact that, when complementarities are weak, agents are roughly indifferent to the degree of coordination in their information. As a result, agent i is much less responsive in her own information choice to the degree to which other agents are direct their search. As a result, the set of equilibria under weak complementarity includes a range of information-search profiles that would be eliminated if agents had stronger strategic incentives.

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