

Online Appendices

Persistent Liquidity Effect and
Long Run Money Demand

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PROOFS

A1. Proof of Proposition 1.

The tilde allocation satisfies market clearing for cash balances and consumption, since they are satisfied in the original equilibrium. Market clearing of bonds is satisfied trivially by construction. The budget constraint of the government is satisfied by construction too, given that the budget constraint for the original equilibrium is satisfied. The tilde allocation solve the problem of the non-traders since their budget constraint is identical. Finally, for traders, at the tilde equilibrium prices and interest rates and chosen initial conditions, the allocation $\{c_t^T, m_t^T\}_{t=0}^\infty$ is budget feasible, and the first order condition for the bond holdings is satisfied. Hence, $\{c_t^T, m_t^T\}_{t=0}^\infty$ solve the trader's problem.

A2. Proof of Proposition 2.

Let $\bar{c} = \lambda \bar{c}^T + (1 - \lambda) \bar{c}^N$ and $\bar{m} = \lambda \bar{m}^T + (1 - \lambda) \bar{m}^N$ be the aggregate steady state values. Using the homotheticity of U we can write $\bar{c}^i = \omega_i \bar{c}$ and $\bar{m}^i = \omega_i \bar{m}$ for $i = T, N$. In this proof, to simplify the notation we also use $\lambda_T = \lambda$ and $\lambda_N = 1 - \lambda$. Let \bar{U}_r and \bar{U}_{rs} be the marginal utilities evaluated at (\bar{c}, \bar{m}) for $r, s = 1, 2$. Using that U is h.o.d. $1 - 1/\gamma$ we have: $\bar{U}_{rs}^i / \bar{U}_1^i = \omega^{-1} \bar{U}_{rs} / \bar{U}_1$ and $\bar{U}_r^i / \bar{U}_1^i = \bar{U}_r / \bar{U}_1$ for $r, s = 1, 2$ and $i = T, N$. We can write the Euler equation of each type i as:

$$\begin{aligned} \frac{\bar{U}_{11}}{\bar{U}_1} \frac{1}{\omega_i} \hat{c}_t^i + \frac{\bar{U}_{12}}{\bar{U}_1} \frac{1}{\omega_i} \hat{m}_t^i &= \mathbb{E}_t \frac{\beta}{\bar{\pi}} \left[\left(\frac{\bar{U}_{11} + \bar{U}_{21}}{\bar{U}_1} \right) \frac{1}{\omega_i} \hat{c}_{t+1}^i + \left(\frac{\bar{U}_{12} + \bar{U}_{22}}{\bar{U}_1} \right) \frac{1}{\omega_i} \hat{m}_{t+1}^i \right] \\ &- \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} \left[\frac{\bar{U}_1 + \bar{U}_2}{\bar{U}_1} \right] \hat{\pi}_{t+1}. \end{aligned}$$

Multiply each expression by $\omega_i \lambda_i / (\lambda_T \omega_T + \lambda_N \omega_N)$ to obtain:

$$\begin{aligned} &\frac{\bar{U}_{11}}{\bar{U}_1} \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \lambda_i \hat{c}_t^i + \frac{\bar{U}_{12}}{\bar{U}_1} \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \lambda_i \hat{m}_t^i \\ &= \mathbb{E}_t \frac{\beta}{\bar{\pi}} \left[\left(\frac{\bar{U}_{11} + \bar{U}_{21}}{\bar{U}_1} \right) \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \lambda_i \hat{c}_{t+1}^i + \left(\frac{\bar{U}_{12} + \bar{U}_{22}}{\bar{U}_1} \right) \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \lambda_i \hat{m}_{t+1}^i \right] \\ &- \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} \left[\frac{\bar{U}_1 + \bar{U}_2}{\bar{U}_1} \right] \frac{\lambda_i \omega_i}{\lambda_T \omega_T + \lambda_N \omega_N} \hat{\pi}_{t+1}. \end{aligned}$$

Adding these expressions for T and N we have:

$$\begin{aligned} & \frac{\bar{U}_{11}}{\bar{U}_1} \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \hat{c}_t + \frac{\bar{U}_{12}}{\bar{U}_1} \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \hat{m}_t \\ = & \mathbb{E}_t \frac{\beta}{\bar{\pi}} \left[\left(\frac{\bar{U}_{11} + \bar{U}_{21}}{\bar{U}_1} \right) \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \hat{c}_{t+1} + \left(\frac{\bar{U}_{12} + \bar{U}_{22}}{\bar{U}_1} \right) \frac{1}{\lambda_T \omega_T + \lambda_N \omega_N} \hat{m}_{t+1} \right] \\ - & \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} \left[\frac{\bar{U}_1 + \bar{U}_2}{\bar{U}_1} \right] \hat{\pi}_{t+1}. \end{aligned}$$

where $\hat{c}_t = \lambda_T \hat{c}_t^T + \lambda_N \hat{c}_t^N$ and $\hat{m}_t = \lambda_T \hat{m}_t^T + \lambda_N \hat{m}_t^N$ are the deviation from steady state of the aggregate consumption and aggregate real balances. Note that

$$\lambda_T \omega_T + \lambda_N \omega_N = \lambda_T \frac{\bar{c}^T}{\bar{c}} + \lambda_N \frac{\bar{c}^N}{\bar{c}} = \frac{\lambda_T \bar{c}^T}{\lambda_T \bar{c}^T + \lambda_N \bar{c}^N} + \frac{\lambda_N \bar{c}^N}{\lambda_T \bar{c}^T + \lambda_N \bar{c}^N} = 1.$$

Moreover feasibility implies that $\hat{c}_t = 0$, hence the Euler equation becomes

$$\frac{\bar{U}_{12}}{\bar{U}_1} \hat{m}_t = \mathbb{E}_t \frac{\beta}{\bar{\pi}} \left[\left(\frac{\bar{U}_{12} + \bar{U}_{22}}{\bar{U}_1} \right) \hat{m}_{t+1} \right] - \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} \left[\frac{\bar{U}_1 + \bar{U}_2}{\bar{U}_1} \right] \hat{\pi}_{t+1}$$

Using the linearized money growth identity in equation (17) to replace $\hat{\pi}_{t+1}$ into the Euler equation we get

$$\frac{\bar{U}_{12}}{\bar{U}_1} \hat{m}_t = \mathbb{E}_t \frac{\beta}{\bar{\pi}} \left[\left(\frac{\bar{U}_{12} + \bar{U}_{22}}{\bar{U}_1} \right) \hat{m}_{t+1} \right] - \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} \left[\frac{\bar{U}_1 + \bar{U}_2}{\bar{U}_1} \right] \left(\hat{\mu}_{t+1} + \frac{\bar{\mu}}{\bar{m}} \hat{m}_t - \frac{\bar{\mu}}{\bar{m}} \hat{m}_{t+1} \right)$$

or

$$\bar{U}_{12} \hat{m}_t + \frac{\beta}{\bar{\pi}^2} [\bar{U}_1 + \bar{U}_2] \frac{\bar{\mu}}{\bar{m}} \hat{m}_t = \mathbb{E}_t \frac{\beta}{\bar{\pi}} [(\bar{U}_{12} + \bar{U}_{22}) \hat{m}_{t+1}] - \mathbb{E}_t \frac{\beta}{\bar{\pi}^2} [\bar{U}_1 + \bar{U}_2] \left(\hat{\mu}_{t+1} - \frac{\bar{\mu}}{\bar{m}} \hat{m}_{t+1} \right)$$

multiplying by \bar{m} and using that in the steady state $\bar{\pi} = \bar{\mu}$ and $\bar{U}_1 = \beta/\bar{\mu} (\bar{U}_1 + \bar{U}_2)$:
(A1)

$$[\bar{U}_{12} \bar{m} + \bar{U}_1] \hat{m}_t = -\mathbb{E}_t \frac{\beta}{\bar{\mu}^2} [\bar{U}_1 + \bar{U}_2] \bar{m} \hat{\mu}_{t+1} + \mathbb{E}_t \frac{\beta}{\bar{\pi}} [\bar{m} (\bar{U}_{12} + \bar{U}_{22}) + \bar{U}_1 + \bar{U}_2] \hat{m}_{t+1}$$

which is identical to the Euler Equation for the aggregate model that is obtained by linearizing (18) around the steady state.

A3. Proof of Proposition 3.

Linearizing (18) around the steady state gives equation (A1). Using the definitions for α and ϕ in equation (21) gives (20).

A4. Sufficient conditions for $0 < \phi < 1$ and $\alpha < 0$

LEMMA 1. Let the utility be given by equation (12). Then $\phi < 1$ and $\alpha < 0$ if

$$\frac{1}{\gamma} < 1 + \frac{1}{\rho} + \frac{1}{r(m/c)} \quad \text{and} \quad \rho < \infty$$

As $\rho \rightarrow \infty$ then $\phi \rightarrow 1$. Finally $\phi > 0$ if

$$\frac{1}{\gamma} < \frac{1}{1+r} \left(1 + \frac{1}{\rho} + \frac{1}{r(m/c)} + r - \frac{1/\rho - 1}{(m/c)} \right) .$$

Proof. Simple algebra using equation (12) gives

$$\begin{aligned} U_2 &= \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{\frac{1-\frac{1}{\gamma}}{1-1/\rho}-1} \frac{1}{\mathcal{A}} m^{-1/\rho} \\ mU_{22} &= \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{\frac{1-\frac{1}{\gamma}}{1-1/\rho}-1} \frac{1}{\mathcal{A}} m^{-1/\rho} \left(-\frac{1}{\gamma} + \frac{1}{\rho} \right) \frac{\frac{1}{\mathcal{A}} m^{1-1/\rho}}{\left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]} \\ &\quad - \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{\frac{1-\frac{1}{\gamma}}{1-1/\rho}-1} \frac{1}{\mathcal{A}} m^{-1/\rho} \left(\frac{1}{\rho} \right) \end{aligned}$$

and

$$\begin{aligned} U_1 &= \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{\frac{1-\frac{1}{\gamma}}{1-1/\rho}-1} c^{-1/\rho} \\ mU_{12} &= \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{\frac{1-\frac{1}{\gamma}}{1-1/\rho}-1} c^{-1/\rho} \left(-\frac{1}{\gamma} + \frac{1}{\rho} \right) \frac{\frac{1}{\mathcal{A}} m^{1-1/\rho}}{\left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]} \end{aligned}$$

thus

$$\phi \equiv (\beta/\bar{\mu}) \left[1 + \frac{\bar{U}_2 + \bar{m}\bar{U}_{22}}{\bar{U}_1 + \bar{m}\bar{U}_{12}} \right] = \frac{1}{1+r} \left[1 + r \frac{\left(1 - \frac{1}{\gamma} \right) r(m/c) - \frac{1}{\rho} + 1}{\left(1 - \frac{1}{\gamma} + \frac{1}{\rho} \right) r(m/c) + 1} \right] .$$

Sufficient conditions for $0 < \phi < 1$ are the following. The condition for $\phi < 1$

requires:

$$\frac{\left(1 - \frac{1}{\gamma}\right) r(m/c) - \frac{1}{\rho} + 1}{\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) + 1} < 1$$

If $\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) + 1 > 0$, this inequality is:

$$0 < \frac{1}{\rho} (1 + rm/c)$$

which holds if $\rho < \infty$. If $\rho = \infty$, then $\phi = 1$. This establishes the condition for $\phi < 1$:

$$\frac{1}{\gamma} < 1 + \frac{1}{\rho} + \frac{1}{r(m/c)} \quad \text{and } \rho < \infty .$$

The condition for $\phi > 0$ requires

$$-1 < r \frac{\left(1 - \frac{1}{\gamma}\right) r(m/c) - \frac{1}{\rho} + 1}{\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) + 1}$$

Since the denominator is positive (whenever $\phi < 1$), this inequality implies

$$-\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) - 1 < r \left[\left(1 - \frac{1}{\gamma}\right) r(m/c) - \frac{1}{\rho} + 1 \right]$$

which, after rearranging terms, gives the second inequality in Lemma 1 .

Finally the condition for $\alpha < 0$ if and only if $\bar{U}_1 + \bar{m}\bar{U}_{12} > 0$. Using equation (13) and the expressions computed above for the utility function equation (12) we have that

$$\frac{U_1 + mU_{12}}{U_1} = \frac{\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) + 1}{1 + r(m/c)}$$

so $U_1 + mU_{12} > 0$ holds if $\left(1 - \frac{1}{\gamma} + \frac{1}{\rho}\right) r(m/c) + 1 > 0$ or, as is immediate to verify, the condition for $\phi < 1$ holds.

Using the steady state equilibrium condition in equation (13) and the definition for the nominal interest rate in equation (19) we can rewrite the condition for $\alpha < 0$ and $\phi < 1$ in terms of exogenous parameters as

$$\frac{1}{\gamma} < 1 + \frac{1}{\rho} + \mathcal{A}^\rho \left(\frac{\bar{\mu}}{\beta} - 1 \right)^{\rho-1} \quad \text{and } \rho < \infty .$$

A5. Proof of Proposition 4.

To simplify the notation we treat the case of $\omega_N = 1$ and also omit the super-index N on n since the problem is only defined for non-traders. Under the stated conditions the value function V is strictly concave and differentiable. The first order condition for this problem is

$$(A2) \quad U_1 \left(y + \tau^N + \frac{n}{\bar{\pi}} - g(n), \frac{n}{\bar{\pi}} + \tau^N \right) = \beta V'(g(n))$$

for all $n \geq 0$. Since V is concave, V' is decreasing and since $U_{11} + U_{12} < 0$ the LHS of the first order condition (A2), for a fixed value $n' = g(n)$, is decreasing in n , and hence $g(\cdot)$ is increasing. The envelope gives:

$$V'(n) = \frac{1}{\bar{\pi}} U_1 \left(y + \tau^N + \frac{n}{\bar{\pi}} - g(n), \frac{n}{\bar{\pi}} + \tau^N \right) + \frac{1}{\bar{\pi}} U_2 \left(y + \tau^N + \frac{n}{\bar{\pi}} - g(n), \frac{n}{\bar{\pi}} + \tau^N \right).$$

Using the first order condition and the envelope evaluated at steady state we obtain:

$$U_1 \left(y + \tau^N - \bar{n} \left(1 - \frac{1}{\bar{\pi}} \right), \frac{\bar{n}}{\bar{\pi}} + \tau^N \right) \left(1 - \frac{\beta}{\bar{\pi}} \right) = \frac{\beta}{\bar{\pi}} U_2 \left(y + \tau^N - \bar{n} \left(1 - \frac{1}{\bar{\pi}} \right), \frac{\bar{n}}{\bar{\pi}} + \tau^N \right)$$

Under the assumption that $U_{12} \geq 0$ and $\bar{\pi} > 1$ it is easy to see that there is a unique steady state \bar{n} satisfying this equation.

We now show that this steady state is globally stable. Suppose not, i.e. that $g'(n) > 1$, and assume that $n_0 > \bar{n}$, then $\lim n_t = \infty$. But notice that V is bounded below, since $V(n) \geq V(0) \geq U(y + \tau^N, \tau^N) / (1 - \beta)$. Additionally we assume that U is bounded above. In this case, since V is concave,

$$V(n_t) \geq V(0) + V'(n_t) n_t$$

and hence as $n_t \rightarrow \infty$ it must be that

$$V'(n_t) = U_1(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, n_t/\bar{\pi} + \tau^N) + U_2(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, n_t/\bar{\pi} + \tau^N) \rightarrow 0.$$

and since $U_2 \geq 0$:

$$\lim_{t \rightarrow \infty} U_1(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, n_t/\bar{\pi} + \tau^N) = 0$$

Since $U_{12} \geq 0$ then

$$U_1(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, n_t/\bar{\pi} + \tau^N) \geq U_1(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, \tau^N)$$

but if $n_{t+1} > n_t$, for $\bar{\pi} > 1$

$$U_1(n_t/\bar{\pi} + y + \tau^N - n_{t+1}, \tau^N) \geq U_1(y + \tau^N, \tau^N) > 0$$

a contradiction. Hence $\lim_t n_t$ must be bounded, and thus $g'(\bar{n}) < 1$.

A6. Proof of Proposition 5.

To simplify the notation we first treat the case of $\omega_N = 1$ and also omit the super-index N on the different variables since the problem is only defined for non-traders.

-Part I. Using $\hat{n}_t = g' \cdot \hat{n}_{t-1}$ for the policy rule gives $\hat{n}_t = \hat{n}_{t-1}/\bar{\pi}$, $\hat{c}_t = (1/\bar{\pi} - g') \hat{n}_{t-1}$ and $\hat{m}_{t+1} = g' \hat{n}_{t-1}/\bar{\pi}$, $\hat{c}_{t+1} = (1/\bar{\pi} - g') g' \hat{n}_{t-1}$. Totally differentiating the Euler equation (6) and using the above policy functions gives a second order ODE with characteristic equation

$$0 = \beta (\varphi_0)^2 + b\varphi_0 + 1,$$

that has $\varphi_0 \equiv g'(\bar{n})$ as its smallest root, where the coefficient b is:

$$(A3) \quad -b \equiv \frac{\bar{\pi} + \frac{\beta}{\bar{\pi}} \left(1 + 2 \frac{\bar{U}_{12}}{U_{11}} + \frac{\bar{U}_{22}}{U_{11}}\right)}{1 + \frac{\bar{U}_{12}}{U_{11}}} > 0.$$

Note that $-b > 0$ under the assumption $U_{11} + U_{12} < 0$ and $\pi > 1$. As an intermediate step for the proof, the next lemma gives the properties of φ_0 as a function of $-b$.

LEMMA 2. The expression for the root that is smaller in absolute value is:

$$(A4) \quad \varphi_0 = \frac{-b - \sqrt{b^2 - 4\beta}}{2\beta}$$

with $0 < \varphi_0 \leq 1$, provided that $-b \geq 1 + \beta$. Moreover, φ_0 is decreasing in $-b$.

Proof of Lemma 2. A real solution requires $b^2 - 4\beta \geq 0$. If $-b \geq 1 + \beta$, then $b^2 \geq (1 + \beta)^2 = 1 + \beta^2 + 2\beta$, and $b^2 - 4\beta \geq (1 - \beta)^2 > 0$. If $-b = 1 + \beta$ then $\varphi_0 = 1$. To see that φ_0 is decreasing in $-b$:

$$\frac{\partial \varphi_0}{\partial (-b)} = \frac{\partial}{\partial (-b)} \left(\frac{-b - \sqrt{(-b)^2 - 4\beta}}{2\beta} \right) = \frac{1}{2\beta} \left(1 - \frac{1}{\sqrt{b^2 - 4\beta}} \right) \leq \frac{1}{2\beta} \left(1 - \frac{1}{1 - \beta} \right) < 0.$$

- Part II. We show that the coefficients of the equation that defines $\varphi_0 \equiv g'(\bar{n})$

are a function of ρ/γ , β , π and m/c . Using $U(c, m) = \left(h(c, m)^{1-1/\gamma} - 1 \right) / (1 - 1/\gamma)$ gives:

$$\frac{U_{22}}{U_{11}} = \frac{h_{22}/h_{11} + r^2 (h_1 h_1) / (-\gamma h h_{11})}{1 + (h_1 h_1) / (-\gamma h h_{11})}, \quad \frac{U_{12}}{U_{11}} = \frac{h_{12}/h_{11} + r (h_1 h_1) / (-\gamma h h_{11})}{1 + (h_1 h_1) / (-\gamma h h_{11})}.$$

Using that h is CES we have

$$\begin{aligned} h_{11} &= \frac{1}{\rho} \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{1/(\rho-1)} c^{-1/\rho-1} \left\{ \frac{c^{1-1/\rho}}{[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho}]} - 1 \right\}, \\ h_{22} &= \frac{1}{\rho} \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{1/(\rho-1)} \frac{1}{\mathcal{A}} m^{-1/\rho-1} \left\{ \frac{\frac{1}{\mathcal{A}} m^{1-1/\rho}}{[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho}]} - 1 \right\}, \\ h_{12} &= \frac{1}{\rho} \left[c^{1-1/\rho} + \frac{1}{\mathcal{A}} m^{1-1/\rho} \right]^{1/(\rho-1)-1} c^{-1/\rho} m^{-1/\rho} / \mathcal{A}. \end{aligned}$$

Thus

$$\frac{h_{22}}{h_{11}} = \left(\frac{c}{m} \right)^2 \quad \text{and} \quad \frac{h_{12}}{h_{11}} = - \left(\frac{c}{m} \right).$$

And using that, as from equation (13), $(\frac{m}{c})^{1-1/\rho} / \mathcal{A} = r m/c$

$$\frac{h_1 h_1}{-h h_{11}} = \frac{\rho}{r (m/c)}.$$

Plugging these expressions into the ones for U_{22}/U_{11} and U_{12}/U_{11} gives

$$(A5) \quad \frac{U_{22}}{U_{11}} = \frac{\left(\frac{c}{m} \right)^2 + r \left(\frac{c}{m} \right) \frac{\rho}{\gamma}}{1 + \frac{1}{r} \left(\frac{c}{m} \right) \frac{\rho}{\gamma}}, \quad \frac{U_{12}}{U_{11}} = \frac{- \left(\frac{c}{m} \right) + \left(\frac{c}{m} \right) \frac{\rho}{\gamma}}{1 + \frac{1}{r} \left(\frac{c}{m} \right) \frac{\rho}{\gamma}}$$

which are functions of $\frac{\rho}{\gamma}$, r , and m/c . Replacing (A5) into (A3) gives:

$$(A6) \quad -b = \frac{\bar{\pi} + \frac{\beta}{\bar{\pi}}(1-x)^2 + x \frac{\tilde{\rho}}{r} (\bar{\pi} + \frac{\beta}{\bar{\pi}}(1+r)^2)}{1-x + x \frac{\tilde{\rho}}{r} (1+r)}$$

where $x \equiv c/m \in (0, 1)$ and $\tilde{\rho} \equiv \rho/\gamma$. The partial derivative of (A6) with respect to $\tilde{\rho}$ gives

$$(A7) \quad \frac{\partial(-b)}{\partial \tilde{\rho}} = \frac{-\frac{x}{r}(x+r)}{(1-x + x \frac{\tilde{\rho}}{r} (1+r))^2} \left(\bar{\pi} - \frac{\beta}{\bar{\pi}}(1+r)(1-x) \right)$$

Noting that at the steady state $\frac{\beta}{\pi}(1+r) = 1$ (by equation (13)) establishes that $-b$ is decreasing in ρ/γ . This, by Lemma 2, implies that the root φ_0 is increasing in ρ/γ .

-Part III. We conclude the proof by showing that $0 < \chi(\bar{m})$ for $\bar{\pi} > 1$ and $m/c > 1$. The proof is in two parts. We first analyze the case of $\gamma = 0$. Then we extend the results for the case of $\gamma > 0$.

Assume $\gamma = 0$. We show that the elasticity of $c(m)$ with respect to m , evaluated at $m = \bar{m}$, is smaller than one, which implies that $\chi(\bar{m}) > 0$. Note by equation (33) that $\frac{m}{c} \frac{\partial c}{\partial m} < 1$ requires $(1 - \frac{c}{m}) \frac{1}{\pi} \leq \varphi_0$. Using equation (A4) this inequality becomes

$$(A8) \quad -b - 2\beta \left(1 - \frac{c}{m}\right) \frac{1}{\pi} - \sqrt{b^2 - 4\beta} \geq 0$$

Using equation (A6) for $\rho/\gamma = 0$ gives

$$(A9) \quad -b = \frac{\pi}{1-x} + \frac{\beta}{\pi} (1-x)$$

where $x \equiv c/m$. We want to show that inequality (A8) holds for $x \in (0, 1)$, i.e. that

$$a(x) \equiv -b(x) - 2\beta(1-x) \frac{1}{\pi} - \sqrt{b(x)^2 - 4\beta} > 0 .$$

This follows because: $a(0) > 0$ and $a'(x) > 0$ for $x \in (0, 1)$. These two inequalities follow from:

$$a(0) = \pi - \frac{\beta}{\pi} - \sqrt{\pi^2 + \left(\frac{\beta}{\pi}\right)^2 + \frac{2\beta}{\pi} - 4\beta}$$

Using that $\pi > 1$, we have

$$a(0) > \pi - \frac{\beta}{\pi} - \sqrt{\pi^2 + \left(\frac{\beta}{\pi}\right)^2 + \frac{2\beta}{\pi} - 4\frac{\beta}{\pi}} = \pi - \frac{\beta}{\pi} - \sqrt{\left(\pi - \frac{\beta}{\pi}\right)^2} = 0 .$$

Finally for $a'(x) > 0$ we have

$$a' = -b' \left[1 + \frac{b}{\sqrt{b(x)^2 - 4\beta}}\right] + 2\frac{\beta}{\pi} = \left(\frac{\pi}{(1-x)^2} - \frac{\beta}{\pi}\right) \left[1 + \frac{b}{\sqrt{b^2 - 4\beta}}\right] + 2\frac{\beta}{\pi} > 0 .$$

We conclude the proof for the $\gamma = 0$ case by showing that $-b > 1 + \beta$ (an assumption in Lemma 2). From equation (A6) with $\tilde{\gamma} = 0$, simple algebra shows

that the inequality $-b > 1 + \beta$ holds if $\pi > 1$ and $m/c > 1$.

These results extend to the case where $\gamma > 0$. As above, the inequality $0 < \chi$ requires $(1 - \frac{c}{m}) \frac{1}{\pi} \leq \varphi_0$. This inequality was shown to hold for $\gamma = 0$. Since φ_0 is increasing in $\tilde{\gamma}$, then it holds *a fortiori* for $\gamma > 0$. The inequality $-b \geq 1 + \beta$ holds, since $-b$ is decreasing in $\tilde{\gamma}$ and $\lim_{\tilde{\gamma} \rightarrow \infty} (-b) = 1 + \beta$.

Finally to see that φ_0 does not change with ω_N and that φ_1 and φ_2 are proportional to ω_N we can use the remaining coefficients of the linearization, whose explicit solution is in Appendix A.A7.

A7. Expressions for the linearized Non-Trader problem

The coefficients for the linearization of the Euler equation of the Non-Trader's problem are:

$$\begin{aligned} \xi_0 &= \frac{[\bar{\pi} \bar{U}_{11} + \frac{\beta}{\bar{\pi}} (\bar{U}_{11} + \bar{U}_{12} + \bar{U}_{21} + \bar{U}_{22})]}{\beta [\bar{U}_{11} + \bar{U}_{21}]}, & \xi_1 &= \frac{\bar{n}^N}{\beta \bar{\pi}}, \\ \xi_2 &= - \left[\frac{\frac{\beta}{\bar{n}} (\bar{U}_1 + \bar{U}_2) + \frac{\beta}{\bar{\pi}} (\bar{U}_{11} + \bar{U}_{12} + \bar{U}_{21} + \bar{U}_{22})}{[\bar{U}_{11} + \bar{U}_{21}]} \right] \frac{\bar{n}^N}{\beta \bar{\pi}}. \end{aligned}$$

Substituting the equilibrium law of motion into the coefficients Non-Trader's problem we have:

$$\begin{aligned} (\varphi_0 - \xi_0) \varphi_0 + \frac{1}{\beta} &= 0, & (\varphi_0 - \xi_0) \varphi_2 &= \xi_1 \left(\frac{\bar{\mu}}{\bar{m}} \right), \\ (\varphi_0 - \xi_0) \varphi_1 + \varphi_1 \Theta + \varphi_2 \kappa &= \xi_1 \zeta + \xi_2 \bar{\Pi}. \end{aligned}$$

The first expression shows that φ_0 is independent of ω_N . Since \bar{n}^N is proportional to ω_N then so are φ_1 and φ_2 .

A8. Proof of Proposition 6.

To simplify the notation we treat the case of $\omega_N = 1$ and also omit the super-index N on \hat{n} since the problem is only defined for non-traders. We try a solution of the form

$$n_t = \varphi_0 \hat{n}_{t-1} + \varphi_1 z_t + \varphi_2 \hat{m}_{t-1}$$

with coefficients φ_0 , φ_1 and φ_2 to be determined. Replacing the hypothesis for inflation and expected inflation the Euler equation:

$$\mathbb{E}_t [\hat{n}_{t+1}] = \xi_0 \hat{n}_t - \frac{1}{\beta} \hat{n}_{t-1} + \xi_1 \left[\left(\frac{\bar{\mu}}{\bar{m}} \right) \hat{m}_{t-1} + \zeta z_t \right] + \xi_2 \bar{\Pi} z_t.$$

Taking expected values on the guess for the solution of the policy:

$$\mathbb{E}_t[\hat{n}_{t+1}] = \varphi_0 \hat{n}_t + \varphi_1 \mathbb{E}_t[z_{t+1}] + \varphi_2 \hat{m}_t = \varphi_0 \hat{n}_t + \varphi_1 \Theta z_t + \varphi_2 \kappa z_t$$

Equating the two terms:

$$\varphi_0 \hat{n}_t + \varphi_1 \Theta z_t + \varphi_2 \kappa z_t = \xi_0 \hat{n}_t - \frac{1}{\beta} \hat{n}_{t-1} + \xi_1 \left[\left(\frac{\bar{\mu}}{\bar{m}} \right) \hat{m}_{t-1} + \zeta z_t \right] + \xi_2 \bar{\Pi} z_t$$

rearranging

$$(\varphi_0 - \xi_0) \hat{n}_t + \varphi_1 \Theta z_t + \varphi_2 \kappa z_t = -\frac{1}{\beta} \hat{n}_{t-1} + \xi_1 \left[\left(\frac{\bar{\mu}}{\bar{m}} \right) \hat{m}_{t-1} + \zeta z_t \right] + \xi_2 \bar{\Pi} z_t$$

Replacing again the guess for the optimal policy in \hat{n}_t :

$$(\varphi_0 - \xi_0) [\varphi_0 \hat{n}_{t-1} + \varphi_1 z_t + \varphi_2 \hat{m}_{t-1}] + \varphi_1 \Theta z_t + \varphi_2 \kappa z_t = -\frac{1}{\beta} \hat{n}_{t-1} + \xi_1 \left[\left(\frac{\bar{\mu}}{\bar{m}} \right) \hat{m}_{t-1} + \zeta z_t \right] + \xi_2 \bar{\Pi} z_t$$

rearranging:

$$\begin{aligned} & (\varphi_0 - \xi_0) \varphi_0 \hat{n}_{t-1} + [(\varphi_0 - \xi_0) \varphi_1 + \varphi_1 \Theta + \varphi_2 \kappa] z_t + (\varphi_0 - \xi_0) \varphi_2 \hat{m}_{t-1} \\ & = -\frac{1}{\beta} \hat{n}_{t-1} + (\xi_1 \zeta + \xi_2 \bar{\Pi}) z_t + \xi_1 \left(\frac{\bar{\mu}}{\bar{m}} \right) \hat{m}_{t-1} \end{aligned}$$

matching coefficients:

$$(\varphi_0 - \xi_0) \varphi_0 = -\frac{1}{\beta} \quad , \quad (\varphi_0 - \xi_0) \varphi_2 = \xi_1 \left(\frac{\bar{\mu}}{\bar{m}} \right) \quad , \quad (\varphi_0 - \xi_0) \varphi_1 + \varphi_1 \Theta + \varphi_2 \kappa = \xi_1 \zeta + \xi_2 \bar{\Pi}$$

That $\varphi_0 = g'(\bar{n})$ can be verified immediately by linearizing the first order condition of the problem for the non-trader with constant inflation $\bar{\pi}$. Then, using Proposition 4, we have $0 < \varphi_0 = g'(\bar{n}) < 1$.

ESTIMATING A TWO-COMPONENT PROCESS FOR μ_t

In this appendix we setup and estimate a two-component process for money growth using monthly US data on M1 for the period 1959.1-2009.9. Let $\{\mu_t\}_{t=1, \dots, T}$ be the sample of observed money growth rates, i.e. the log change in the levels of the nominal money stock. We assume that it is the sum of two unobserved, independent AR(1) processes (with a zero mean) of the form:

$$\mu_{i,t} = \theta_i \mu_{i,t-1} + \epsilon_{i,t}$$

where $i = 1, 2$, the innovations $\{\epsilon_{i,t}\}$ are i.i.d., through time and independent of each other, normally distributed with mean zero and variance σ_i^2 . We have then $\mu_t = \mu_{1,t} + \mu_{2,t}$ for $t = 1, 2, \dots$ thus $\mu_{1,t} + \mu_{2,t} = \frac{\epsilon_{1,t}}{1-\theta_1 L} + \frac{\epsilon_{2,t}}{1-\theta_2 L}$. Being the sum of two AR(1) processes then

$$\mu_t = \phi_1 \mu_{t-1} + \phi_2 \mu_{t-2} + \epsilon_t + \xi \epsilon_{t-1}$$

is an ARMA(2,1) process where ϕ_1, ϕ_2 are the autoregressive parameters and ξ the MA parameter where σ is the standard deviation of ϵ . We next derive the mapping between $\sigma, \phi_1, \phi_2, \xi$ and the original parameters $\sigma_1, \sigma_2, \theta_1, \theta_2$. It is immediate that $\phi_1 = \theta_1 + \theta_2$ while $\phi_2 = -(\theta_1 \theta_2)$.

Let $y_t \equiv (1 - \phi_1 L - \phi_2 L^2) \mu_t = \epsilon_t + \xi \epsilon_{t-1}$, so that $var(y_t) = \sigma^2(1 + \xi^2)$ and $cov(y_t, y_{t-1}) = \xi \sigma^2$. Likewise, using that $\epsilon_t + \xi \epsilon_{t-1} = \epsilon_{1,t} + \epsilon_{2,t} - \theta_2 \epsilon_{1,t-1} - \theta_1 \epsilon_{2,t-1}$ we have $var(y_t) = \sigma_1^2(1 + \theta_2^2) + \sigma_2^2(1 + \theta_1^2)$ and $cov(y_t, y_{t-1}) = -(\theta_1 \sigma_2^2 + \theta_2 \sigma_1^2)$ which gives a 2 dimensional linear system for σ_1 and σ_2

$$\begin{bmatrix} 1 + \theta_2^2 & 1 + \theta_1^2 \\ -\theta_2 & -\theta_1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \end{bmatrix} = \begin{bmatrix} (1 + \xi^2) \sigma^2 \\ \xi \sigma^2 \end{bmatrix}.$$

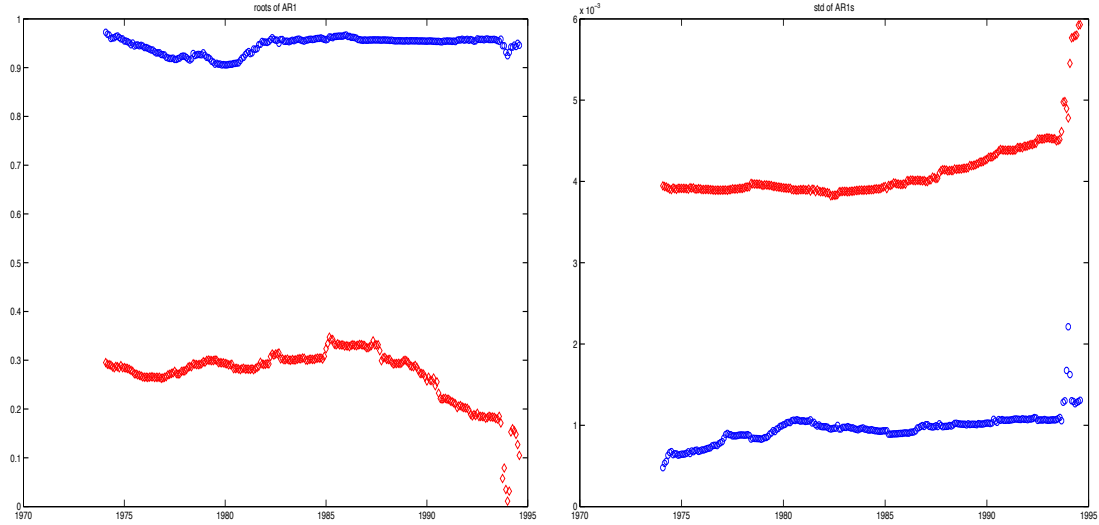
We estimate the ARMA parameters $\sigma, \phi_1, \phi_2, \xi$ using the Kalman filter following Hevia (2008) and map them into the original parameters $\sigma_1, \sigma_2, \theta_1, \theta_2$ using the procedure outlined above. The estimates are reported in Table B1.

TABLE B1—MAXIMUM LIKELIHOOD ESTIMATES OF MONETARY GROWTH

θ_1	θ_2	σ_1	σ_2
0.949	0.115	0.00104	0.00497

Growth rate of $M1$ based on seasonally adjusted data from 1959 to 2010.

Figure B1 reports the estimation results from a set of estimates for these same parameter that use a rolling time window of 30 years. The date on the horizontal axis denotes the sample midpoint. The objective is to scan the sample period to spot qualitative structural changes in the time series behavior of the money growth time series. The estimated autoregressive coefficients appear quite stable over the whole sample period (left panel), with a tendency for the short term component of the series, whose half-life over the whole sample is around 10 days, to become even less persistent (and the variance of the associated innovations increases slightly) since the beginning of the 80s. The process appears less stable in the very last samples (ending in 2008-2009) due to the large innovations to money growth (in the order of 4% per month), that occurred during the financial crises.

AR(1) roots θ_1, θ_2 Standard deviations: σ_1, σ_2 FIGURE B1. ESTIMATES $ARMA(2, 1)$ FOR MONEY GROWTH: 30 YEAR ROLLING WINDOWS ON 1959-2009

Note: Monthly data on M1 growth rate based on seasonally adjusted data (Source is FRED). The observations for october-november 2001, in which months the growth rate of money spikes up by $\pm 4\%$ were replaced by their projected forecast. The date on the horizontal axis is the sample midpoint. The AR(1) process with the more persistent root is plotted in blue.

RELATION TO STICKY-PRICE MODELS (WITH NO CAPITAL)

This section shows that the liquidity effect can be produced in a canonical New Keynesian (NK) model (with no capital) if the intertemporal substitution elasticity is sufficiently small (as shown by Gali (2002)). But requiring that the model also features a unitary long run income elasticity (an important feature to fit the low frequency data), implies that this elasticity is 1 (due to the separability of money and consumption assumed by the NK model, see below), so that the standard model, adapted to fit the low frequency facts, is unable to generate a liquidity effect.

Consider the following long run features of an economy: (1) a unitary income elasticity of money demand and (2) the stylized facts along a BGP (i.e. a constant real interest rate, constant leisure, and constant growth of real wages, real balances and consumption). Both features are desirable to connect the low and high frequency data within the same model. To have a model featuring (1) and (2) the agents' preferences must satisfy

(C1)

$$U(c, m, l) = \begin{cases} \log h(c, m) + v(l) \\ \frac{h(c, m)^{1-\gamma}}{1-\gamma} \cdot v(l) \end{cases} \quad \text{with } h(c, m) > 0 \text{ homogenous of degree 1 in } c, m.$$

These preferences correspond to the first and second case (equations 10 and 11) in Andres, David Lopez-Salido and Valles (2002). Note that the preference satisfy, for any fixed l , the BGP requirement that U_c/U_l is homogenous of degree 1 in (c, m) and that U_m/U_c is homogenous of degree 0 in (c, m) . The first condition ensures consistency with a unit income elasticity, the second that the ratio m/c is a function of the interest rate (see the discussion in Section 6 of Chari, Kehoe and McGrattan (1996)).

Some parametric conditions must be satisfied to have a liquidity effect. Typically these imply a low intertemporal substitution elasticity in a model without capital (a high σ in Galí (2002)). To see this note that from the money demand condition $M/(PY) = R^{-\eta}$ the presence of the liquidity effect means that R fall if M increases. To have this effect it must be that the denominator PY does not increase much as M increases. Now if P is “fixed” because of sticky prices, this requires that Y does not increase much; this condition obtains if the intertemporal substitution elasticity is low.

Imposing requirements (1) and (2) in the NK models (without capital) implies that preferences must either be of the first type (log-log), but then this gives no liquidity effect given the low substitution elasticity of the log case ($\sigma = 1$, see Galí (2002)). One must then use the non-separable specification. Of course non-separability implies a different set of equations than the ones used in NK models, where the money balances do not affect the determination of real variables. Andres, David Lopez-Salido and Valles (2002) discuss the case of non-separable preferences (their utility function 11, section 4.2) and find that no parametrization allows them to produce a liquidity effect on impact (see their figure 2) .

ASSESSING THE PRECISION OF OUR APPROXIMATE SOLUTION

In the main body of the paper we solved the model analytically by using an approximate linearized version of the agents’ Euler equations. This appendix explores the accuracy of our approximation by comparing the analytical solution with the numerical solution of the model original non-linear equations.¹⁶ In this section we compute IRF’s to monetary shocks solving numerically the fully non-linear version of the model. We find that for parameters of interest the linear approximations are quite accurate. In this section we set $\omega = 1$ and explore the effect of segmentation by varying λ .

D1. Non-linear impulse responses with iid shock

To compute the non-linear impulse responses, consider the perfect-foresight version of the model. At $t = 0$, there is an unanticipated, once-and-for-all change

¹⁶Roberto Robatto provided excellent research assistance with this computation.

in the growth rate of money supply:

$$\frac{\mu_0 - \bar{\mu}}{\bar{\mu}} = 1\%$$

Then, we guess that the path of inflation is given by the equilibrium in the linear version of the model:

$$\pi_t = \begin{cases} \bar{\pi} + 1\% & \text{for } t = 0 \\ \bar{\pi} & \text{for } t \geq 1 \end{cases}$$

so you can compute a guess for the aggregate path of money supply m_t , using equation (2).

We start by solving the problem of non-traders: their optimal solution is described by equation (6). Using a shooting algorithm, it is possible to compute the path of money demand $\{m_t^N\}_{t=0}^{\bar{T}}$ and of consumption $\{c_t^N\}_{t=0}^{\bar{T}}$, for a \bar{T} which is large enough. Here is an idea of how the algorithm works:

- the initial level of real money for non-traders is a state variable, and it is given by $m_0^N = \frac{\bar{m}}{\pi_0}$;
- we guess an initial level of consumption, $c_0^N < \bar{c}$
- for every $t \geq 0$, given knowledge of (m_t^N, c_t^N) , we recursively do the following steps
 - using money m_t^N and consumption c_t^N and equation (2), we compute money tomorrow m_{t+1}^N ;
 - using the Euler equation (6), we compute consumption tomorrow c_{t+1}^N ;
 - with knowledge of (m_{t+1}^N, c_{t+1}^N) , we move to $t + 1$;
- when we reach $t = \bar{T}$, we check whether the level of consumption $c_{\bar{T}}^N$ is “close enough” to the steady-state level (up to some tolerance)
 - if yes, the algorithm is concluded;
 - if not, depending on whether $c_{\bar{T}}^N \geq \bar{c}$, we increase or decrease the initial guess for consumption c_0^N , repeating all the above steps

The outcome of this algorithm is a sequence of consumption and money demand for non-traders $\{c_t^N\}_{t=0}^{\bar{T}}$ and $\{m_t^N\}_{t=0}^{\bar{T}}$. Using market clearing in the goods and money market, we then compute a guess for consumption and money demand of traders, that we denote $\{\tilde{c}_t^N\}_{t=0}^{\bar{T}}$ and $\{\tilde{m}_t^N\}_{t=0}^{\bar{T}}$:

$$\tilde{c}_t^N = \frac{y - (1 - \lambda) c_t^N}{\lambda}$$

$$\tilde{m}_t^T = \frac{m_t - (1 - \lambda) m_t^N}{\lambda}$$

Then, we compute the nominal interest rate using:

$$r_t = \frac{U_2(\tilde{c}_t^T, \tilde{m}_t^T)}{U_1(\tilde{c}_t^T, \tilde{m}_t^T)}$$

and the transfers to traders $\{\tau_t^T\}_{t=0}^{\bar{T}}$, using the monetary-fiscal policy described in Section IV.B.

We then perform another shooting algorithm similar to the one described above, but for traders. We guess an initial value c_0^T and given the path of transfers $\{\tau_t^T\}_{t=0}^{\bar{T}}$, the path of money $\{\tilde{m}_t^T\}_{t=0}^{\bar{T}}$, the path of interest rate $\{r_t\}_{t=0}^{\bar{T}}$ and the path of inflation $\{\pi_t\}_{t=0}^{\bar{T}}$, we compute the path of consumption $\{c_t^T\}_{t=0}^{\bar{T}}$ that satisfies the Euler equation equation (10). The state variable in this case is wealth: the initial wealth w_0 and, for all $t \geq 0$, the wealth next period w_{t+1} , are computed using equation (7).

We now have a path of aggregate consumption:

$$c_t = \lambda c_t^T + (1 - \lambda) c_t^N$$

If $c_t = y$ for all t (up to some tolerance), we have found the solution. If $c_t \geq y$, we change the initial guess for inflation: if $c_t > y$ we increase the price level in time t , and vice versa.

D2. Non-linear impulse responses with AR(1) shock

The procedure is very similar to the case of iid shock. Let us just emphasize that, for comparison with the impulse-responses of the linear model, we have a shock to the growth rate of money such that the price level goes up by 1% in the *linear* solution; then, given the same shock to the growth rate of money supply, we compute the non-linear solution (so, in the non-linear solution, the increase in the price level on impact is not exactly 1%), using the same algorithm described above.

D3. Results

We now present the results of the non-linear model: we compare them with the results in the paper, and we discuss the sources of error in the linearized solution.

Since consumption and money supply are exogenous, finding the equilibrium is equivalent to finding a path of nominal interest rates and a path of prices¹⁷ that support the market clearing condition in both the goods market and the money

¹⁷Or, equivalently, a path of nominal interest rates and a path of inflation rates.

market, for all t . Therefore, there are two sources of approximation error in the linearized solution: an error in the path of nominal interest rate, and an error in the path of prices.

All the results are computed setting the intertemporal elasticity of substitution $\gamma = 1/4$, while we vary the parameters λ (fraction of traders), θ (persistence of the shock to the growth rate of money supply) and ρ (elasticity of substitution between consumption and money in the utility function).

To understand the error in the path of interest rates, Figure D1 plots the response of the interest rate to a money supply shock for both the linear and the non-linear model, for various parameters configuration that correspond to Figure 1, 2 and 3 in the paper. The solid line is the response of interest rate in the linearized solution, and the dotted line is the response of interest rate in the non-linear solution. To understand the error in the path of prices, Figure D2 plots instead:

$$100 \times \{\log (P_t [\text{linearized solution}]) - \log (P_t [\text{non-linear solution}])\}$$

because this is more informative for this error¹⁸.

To better understand the results, we first compute the response to a persistent money supply shock in the case with no segmentation ($\lambda = 1$), that corresponds to Figure 3 in the paper. Interest rates are plotted in the top panel of Figure D1, while the difference in prices is plotted in the top panel of Figure D2. We consider four different possibilities for the persistence of the money supply shock ($\theta = 0$, $\theta = 0.5$, $\theta = 0.8$ and $\theta = 0.99$, that correspond to an half life of zero months, one months, three months and ten years), while we fix $\rho = 1/2$. The impulse is a positive shock to the growth rate of money supply such that the price level increases 1% in the linearized solution.

For $\theta = 0$, the two solutions are identical: there is a once-and-for-all increase in money on impact, that is compensated by an equal increase in prices. For $\theta > 0$, the approximation error is non-zero but small. The top panel of Figure D1 shows that the response of interest rate in the linear and non-linear solutions are almost identical. From the top panel of Figure D2, the largest price difference is in the order of magnitude of 0.004%.

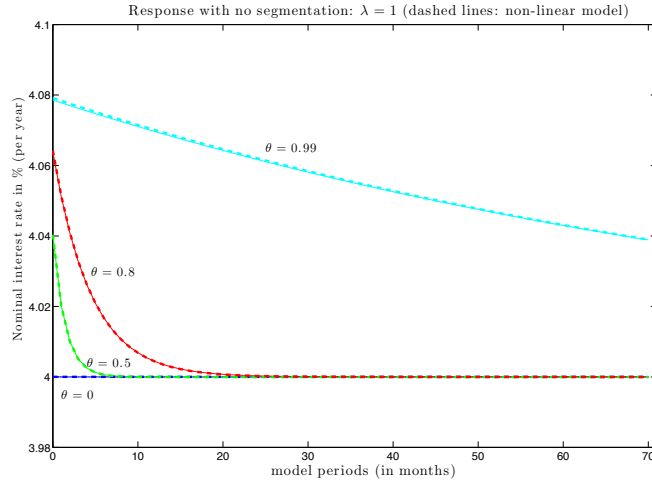
Second, we compute the response to a persistent money supply shock when there are both traders and non-traders in the economy (we set the fraction of traders $\lambda = 0.25$), that corresponds to Figure 2 in the paper. We again consider four possible values of θ and we set $\rho = 1$; the impulse is a positive shock to the growth rate of money supply such that the price level increases 1% in the linearized solution. The results are plotted in the mid panels of Figure D1 and D2.

¹⁸The path of prices in the linear and non-linear solutions are almost identical, and overlap each other almost exactly if we plot them. So the best way to highlight the effect of changing the parameters on this approximation error is to plot the (log) difference between the two price levels.

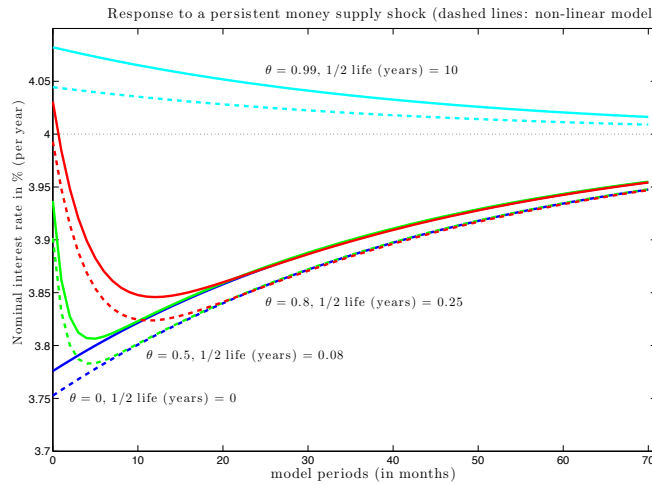
Fixing θ , the comparison between the response with $\lambda = 1$ (top panel) and with $\lambda = 0.25$ (mid panel) allows to identify the role of market segmentation on the approximation errors. With $\lambda = 0.25$, the approximation error on interest rate widens substantially. But the result that market segmentation creates a liquidity effect for shorter half-life is confirmed using the non-linear solution. Moreover, for a given set of parameters, the linearized solution underestimates the effect of market segmentation on interest rates. With respect to prices, all errors are shifted upward but overall their magnitude is unchanged in absolute value because they move from slightly negative to slightly positive.

Third, we fix $\theta = 0$ (no persistence in the money supply shock) and we vary the elasticity of substitution ρ between consumption and money in the utility function (we set $\rho = 1/2$, $\rho = 1/4$ and $\rho = 1/8$), similarly to Figure 1 in the paper. The impulse is a once-and-for-all 1% increase in money supply. The results are plotted in the bottom panel of Figure D1 and Figure D2. The response labelled “ $\rho = 1/2$ ” is the same as the response labelled “ $\theta = 0$ ” in the mid panel, so a comparison between the mid panel and the bottom panel allows to identify the effect of changing ρ on the approximation errors.

The approximation error in the response of interest rates is not affected much by decreasing ρ . On the contrary, decreasing ρ has a substantial effect on the approximation error of prices on impact, but such error decrease rapidly and it is anyway no larger than 3 basis points. With respect to the analysis in the paper about the liquidity effect with an iid shock, the bottom panel of Figure D1 confirms that higher values of ρ imply both a larger effect on impact and faster convergence.



Solid lines: responses of the linearized model; dashed lines: responses of the non-linear model. The shock is an unanticipated increase in money causing a 1% increase of the price level on impact in the linearized solution. The other parameters are: $\gamma = 1/4$, $\lambda = 1$, $\rho = 1/2$.



Solid lines: responses of the linearized model; dashed lines: responses of the non-linear model. The shock is an unanticipated increase in money causing a 1% increase of the price level on impact in the linearized solution. The other parameters are: $\gamma = 1/4$, $\lambda = 0.25$, $\omega = 1$, $\rho = 1/2$.

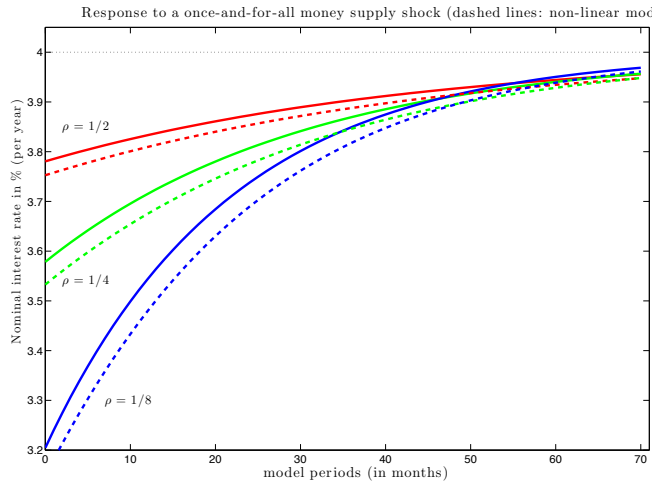
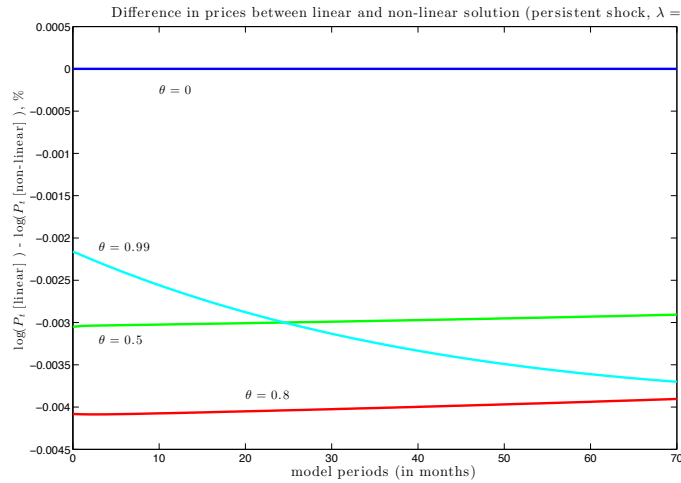
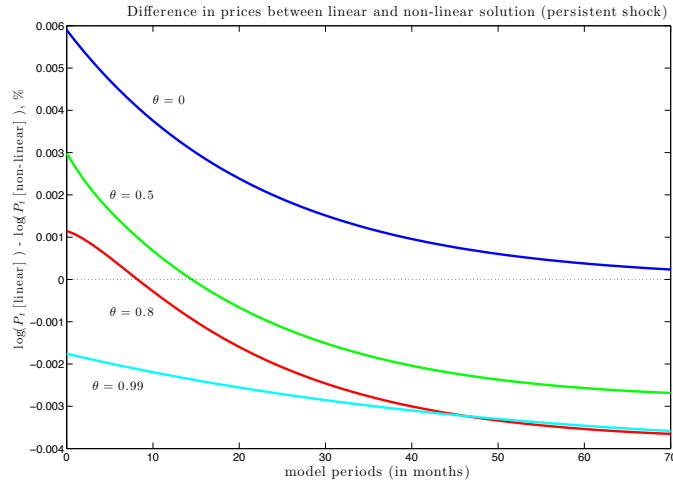


FIGURE D1. RESPONSE OF NOMINAL INTEREST RATE TO A MONEY SUPPLY SHOCK

Solid lines: responses of the linearized model; dashed lines: responses of the non-linear model. The shock is an unanticipated once-and-for-all 1% increase in money supply. The other parameters are: $\gamma = 1/4$, $\omega = 1$, $\lambda = 0.25$, $\theta = 0$.



The shock is an unanticipated increase in money causing a 1% increase of the price level on impact in the linearized solution. The other parameters are: $\gamma = 1/4$, $\lambda = 1$, $\rho = 1/2$.



The shock is an unanticipated increase in money causing a 1% increase of the price level on impact in the linearized solution. The other parameters are: $\gamma = 1/4$, $\lambda = 0.25$, $\rho = 1/2$.

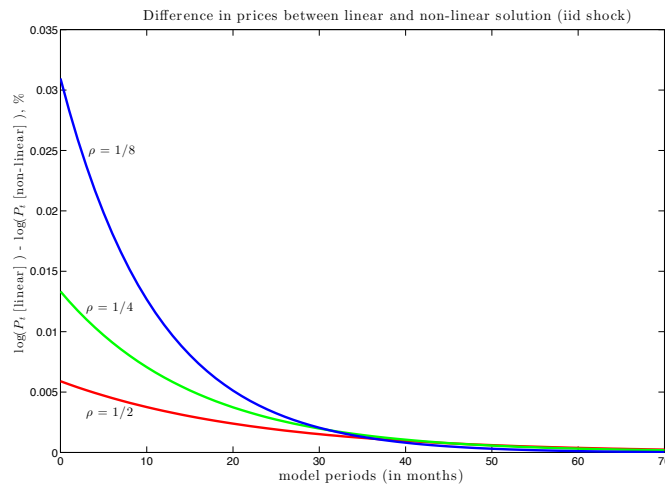


FIGURE D2. PRICE LEVEL APPROXIMATION ERROR

The shock is an unanticipated increase in money causing a 1% increase of the price level on impact. The other parameters are: $\gamma = 1/4$, $\lambda = 0.25$, $\theta = 0$.