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Real Wage Inequality
Web Appendix

Here I present the details of the model described in Section 4. I assume that each city is a competitive economy that produces a single output good y which is traded on the international market, so that its price is the same everywhere and set equal to 1. Like in Roback, I abstract from labor supply decisions and I assume that each worker provides one unit of labor, so that local labor supply is only determined by workers' location decisions. The indirect utility of skilled workers in city c is assumed to be

$$U_{Hic} = w_{Hc} - r_c + A_{Hc} + e_{Hic} \tag{1}$$

where w_{Hc} is the nominal wage in the city; r_c is the cost of housing; A_{Hc} is a measure of local amenities. The random term e_{Hic} represents worker i idiosyncratic preferences for location c . A larger e_{Hic} means that worker i is particularly attached to city c , holding constant real wage and amenities. For example, being born in city c or having family in city c may make city c more attractive to a worker. Similarly, the indirect utility of unskilled workers is

$$U_{Lic} = w_{Lc} - r_c + A_{Lc} + e_{Lic} \tag{2}$$

In equations 1 and 2, skilled and unskilled workers in a city compete for housing in the same housing market and therefore face the same price of housing. This allows a shock to one group to be transmitted to the other group through its effect on housing prices.¹ While they have access to the same local amenities, different skill groups do not need to value these amenities equally: A_{Hc} and A_{Lc} represent the skill-specific value of local amenities.

Assume that there are two cities—Detroit (city a) and San Francisco (city b)—and a fixed number of workers is divided between the two cities. Tastes for location can vary by

¹It is easy to relax this assumption by assuming some residential segregation by skill level within a city.

skill group. Specifically, skilled workers' and unskilled workers' relative preferences for city a over city b are, respectively

$$e_{Hia} - e_{Hib} \sim U[-s_H, s_H] \quad (3)$$

and

$$e_{Lia} - e_{Lib} \sim U[-s_L, s_L] \quad (4)$$

The parameters s_H and s_L characterize the importance of idiosyncratic preferences for location and therefore the degree of labor mobility. If s_H is large, for example, it means that preferences for location are important for skilled workers and therefore their willingness to move to arbitrage away real wage differences or amenity differences is limited. On the other hand, if s_H is small, preferences for location are not very important and therefore skilled workers are more willing to move in response to differences in real wages or amenities. In the extreme, if $s_H = 0$ skilled workers' mobility is perfect.

A worker chooses city a if and only if $e_{ia} - e_{ib} > (w_b - r_b) - (w_a - r_a) + (A_b - A_a)$. In equilibrium, the marginal worker needs to be indifferent between living in Detroit and San Francisco. This implies that skilled workers' labor supply is upward sloping, with the slope that depends on s . For example, the supply of skilled workers in San Francisco is:

$$w_{Hb} = w_{Ha} + (r_b - r_a) + (A_a - A_b) + s_H \left(\frac{N_{Hb} - N_{Ha}}{N} \right) \quad (5)$$

where N_{Hb} is the log of the number of skilled workers hired in San Francisco and $N = N_{Ha} + N_{Hb}$. If idiosyncratic preferences for location are not very important (s_H is small), then workers are very mobile and the supply curve is relatively flat. If idiosyncratic preferences for location are very important (s_H is large), then workers are rather immobile and the supply curve is relatively steep. Moreover, an increase in the real wage in Detroit, or an improvement in relative amenities shifts back the labor supply curve in San Francisco.²

For simplicity, I focus on the case where skilled and unskilled workers in the same city work in different firms. This amounts to assuming away imperfect substitution between skilled and unskilled workers. This assumption simplifies the analysis, and it is not crucial

²An important difference between the Rosen-Roback setting and this setting is that in Rosen-Roback, all workers are identical, and always indifferent across locations. In this setting, workers differ in their preferences for location. While the marginal worker is indifferent between locations, here there are inframarginal workers who enjoy economic rents. These rents are larger the smaller the elasticity of local labor supply.

(Moretti, 2010). The production function for firms in city c that use skilled labor is Cobb-Douglas with constant returns to scale: $\ln y_{Hc} = X_{Hc} + hN_{Hc} + (1 - h)K_{Hc}$, where K_{Hc} is the log of capital and X_{Hc} is a skill and city-specific productivity shifter. Firms are assumed to be perfectly mobile. If firms are price takers and labor is paid its marginal product, labor demand for skilled labor in city c is

$$w_{Hc} = X_{Hc} - (1 - h)N_{Hc} + (1 - h)K_{Hc} + \ln h \quad (6)$$

The labor market for unskilled workers is similar. I assume that there is an international capital market, and that capital is infinitely supplied at a given price i .³

Each worker consumes one unit of housing, so that demand for housing is determined by the number of skilled and unskilled workers in a city. Specifically, the the local demand for housing is the sum the demand of skilled workers and the demand of unskilled workers. For example, in city b :

$$r_b = \frac{(2s_H s_L)}{(s_H + s_L)} - \frac{(2s_H s_L)(N_{Hb} + N_{Lb})}{N(s_H + s_L)} - \frac{s_L(w_{Ha} - w_{Hb} - r_a)}{(s_L + s_H)} - \frac{s_H(w_{La} - w_{Lb} - r_a)}{(s_L + s_H)} \quad (7)$$

To close the model, I assume that the supply of housing is

$$r_c = z + k_c N_c \quad (8)$$

where $N_c = N_{Hc} + N_{Lc}$ is the number of housing units in city c , which is the same as the number of workers. The parameter k_c characterizes the elasticity of the supply of housing. I assume that this parameter is exogenously determined by geography and local land regulations. In cities where geography and regulations make it easy to build new housing, k_c is small. In the extreme case where there are no constraints to building new houses, the supply curve is horizontal, and k_c is zero. In cities where geography and regulations make it difficult to build new housing, k_c is large. In the extreme case where it is impossible to build new houses, the supply curve is vertical, and k_c is infinite.⁴

³In equilibrium demand for capital is equal to its supply and marginal product of capital is the same for firms that use skilled labor and those that use unskilled labor: $X_{Hc} - hK_{Hc} + hN_{Hc} + \ln(1 - h) = \ln i X_{Lc} - hK_{Lc} + hN_{Lc} + \ln(1 - h) = \ln i$.

⁴A limitation of equation 8 is housing production does not involve the use of any local input. Roback (1982) and Glaeser (2008), among others, discuss spatial equilibrium in the case where housing production

In period 1, the two cities are assumed to be identical. Equilibrium in the labor market is obtained by equating equations 5 and 6 for each city. Equilibrium in the housing market is obtained by equating equations 7 and 8. I consider two scenarios for period 2. In the first scenario, the relative demand of skilled workers increases in one of the two cities. In the second scenario, the relative supply of skilled workers increases in one of the two cities.

Increase in the Relative Demand of Skilled Labor

Here I consider the case where the productivity of skilled workers increases relative to the productivity of unskilled workers in San Francisco. Nothing happens to the productivity of unskilled workers in San Francisco and the productivity of skilled and unskilled workers in Detroit. In other words, the relative demand for skilled labor increases in San Francisco. The amenities in the two cities are identical and fixed. Formally, I assume that in period 2, the productivity shifter for skilled workers in San Francisco is higher than in period 1: $X_{Hb2} = X_{Hb1} + \Delta$, where $\Delta > 0$ represents a positive, localized, skill-biased productivity shock. I have added subscripts 1 and 2 to denote periods 1 and 2. The dot-com boom experienced by the San Francisco Bay Area is arguably an example of such a localized skill biased shock. Driven by the advent of the Internet and the agglomeration of high tech firms in the area, the demand for skilled workers increased significantly (relative to the demand for unskilled workers) in San Francisco in the second half of the 1990s.⁵

Because skilled workers in San Francisco have become more productive, their nominal wage increases by an amount Δ/h , proportional to the productivity increase. Attracted by this higher productivity, some skilled workers leave Detroit and move to San Francisco. Following this inflow of skilled workers, the cost of housing in San Francisco increases by

$$r_{b2} - r_{b1} = \frac{s_L N k_b \Delta}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \geq 0 \quad (9)$$

In Detroit, the cost of housing declines by the same amount because of out-migration.

 involves the use of local labor and other local inputs. Moreover, equation 8 ignores the durability of housing. Glaeser and Gyourko (2001) point out that once built, the housing stock does not depreciate quickly and this introduces an asymmetry between positive and negative demand shocks. In particular, when demand declines, the quantity of housing cannot decline, at least in the short run.

⁵Beaudry, Doms and Lewis (2008) argue that over the past 30 years, technological change resulted in increases in the productivity of skilled workers in cities that already had many skilled workers. These cities also happen to be cities with a higher than average initial share of college graduates and cost of housing. See also Berry and Glaeser (2005).

In San Francisco, real wages of skilled workers increase by

$$(w_{Hb2} - r_{b2}) - (w_{Hb1} - r_{b1}) = \frac{k_a N s_H + k_b N s_H + k_a N s_L + 2s_H s_L}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta \geq 0 \quad (10)$$

It is easy to see that, because of the increased cost of housing, the increase in real wages is smaller than the increase in nominal wages Δ/h . Moreover, this increase in the real wage of skilled workers is larger the more elastic is housing supply in San Francisco (small k_b). Intuitively, a more elastic housing supply implies a smaller increase in housing prices in San Francisco, and therefore a larger increase in real wage, for a given increase in nominal wage. The increase in the real wage of skilled workers is also larger the smaller the elasticity of local labor supply of skilled workers (large s_H). Intuitively, lower elasticity of labor supply implies less mobility. With less mobility, a larger fraction of the benefit of the productivity shocks is capitalized in real wages. In the extreme case of no mobility, ($s_H = \infty$), the entire productivity shock is capitalized in the real wage of skilled workers. The increase in the real wage of skilled workers is larger the larger the elasticity of local labor supply of skilled workers (small s_L). A higher elasticity of labor supply of unskilled workers implies that a larger number of unskilled workers move out in response to the inflow of skilled workers, so that the increase in housing costs is more limited.

In Detroit nominal wages don't change and housing costs decline, so that real wages for skilled workers increase by

$$(w_{Ha2} - r_{a2}) - (w_{Ha1} - r_{a1}) = \frac{s_L k_a N}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta \geq 0 \quad (11)$$

Although the shock has increased productivity only in one city, the equilibrium real wages of skilled workers increase in both cities because of mobility. By comparing equation 10 with 11, it is easy to see that the increase in real wages in the city directly affected by the productivity shock (San Francisco) is larger than the increase in real wages in the city not affected by the productivity shock (Detroit): $(w_{Hb2} - r_{b2}) - (w_{Hb1} - r_{b1}) \geq (w_{Ha2} - r_{a2}) - (w_{Ha1} - r_{a1})$. This is not surprising. While labor mobility causes real wages to increase in Detroit following a shock in San Francisco, real wages are not fully equalized because mobility is not perfect and only the marginal worker is indifferent between the two cities in equilibrium. With perfect mobility ($s_H = 0$), real wages are completely equalized.

What happens to the wage of unskilled workers? Because their productivity is fixed, their nominal wage does not change. However, housing costs increase in San Francisco and decline in Detroit. As a consequence, the real wage of unskilled workers in San Francisco decreases by

$$(w_{Lb2} - r_{b2}) - (w_{Lb1} - r_{b1}) = -\frac{s_L N k_b}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta \leq 0 \quad (12)$$

Effectively, unskilled workers compete for scarce housing with skilled workers, and the inflow of new skilled workers in San Francisco hurts inframarginal unskilled workers through higher housing costs. Marginal unskilled workers leave San Francisco, since their real wage is higher in Detroit. Inframarginal unskilled workers (those who have a strong preference for San Francisco over Detroit) opt to stay in San Francisco, even if their real wage is lower. For the same reason, the real wage and utility of inframarginal unskilled workers in Detroit increases:

$$(w_{La2} - r_{a2}) - (w_{La1} - r_{a1}) = \frac{s_L N k_a}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta \geq 0 \quad (13)$$

The equilibrium number of skilled workers increases in San Francisco, while the equilibrium number of unskilled workers decreases. Changes in employment in Detroit are exactly specular, by assumption. On net, the overall population of San Francisco increases because the number of skilled workers who move in is larger than the number of unskilled workers who leave.⁶

An assumption of this model is that skilled and unskilled workers are employed by different firms, so that the labor market is segregated by skill within a city. This assumption

⁶In particular, the number of skilled workers in San Francisco increases by

$$N_{Hb2} - N_{Hb1} = \frac{\Delta N((k_a + k_b)N + 2s_L)}{2h(k_a N(s_H + s_L) + k_b N(s_H + s_L) + 2s_H s_L)} \geq 0 \quad (14)$$

The number of unskilled workers declines by

$$N_{Lb2} - N_{Lb1} = -\frac{N^2(k_a + k_b)}{2h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta \leq 0 \quad (15)$$

San Francisco population increases by

$$(N_{Hb2} + N_{Lb2}) - (N_{Hb1} + N_{Lb1}) = \frac{\Delta N s_L}{h(k_a N(s_H + s_L) + k_b N(s_H + s_L) + 2s_H s_L)} \geq 0 \quad (16)$$

effectively rules out imperfect substitutability between skilled and unskilled labor. In a more general setting, skilled and unskilled workers work in the same firm. The qualitative results generalize, but the equilibrium depends on the degree of imperfect substitution between skilled and unskilled labor. Specifically, complementarity between skilled and unskilled workers implies that the marginal product of unskilled workers increases in the number of skilled workers in the same firm. Thus, the inflow of skilled workers in city b caused by the increase in their productivity endogenously raises the productivity of unskilled workers in city b . As a consequence, the real wage of unskilled workers declines less than in the case described above. This mitigates the negative effect on the welfare of unskilled workers in city b and it reduces the number of unskilled workers who leave the city.

Increase in the Relative Supply of Skilled Labor

In the case of demand pull described above, the number of skilled workers in San Francisco increases because the relative demand of skilled workers increases. I now turn to the opposite case, where the number of skilled workers in San Francisco increases because the relative supply of skilled workers in San Francisco increases.

Specifically, I consider what happens when San Francisco becomes relatively more desirable for skilled workers compared to Detroit. I assume that in period 2, the amenity level increases for skilled workers in San Francisco: $A_{Hb2} = A_{Hb1} + \Delta'$, where $\Delta' > 0$ represents the improvement in the amenity. I assume that the productivity of both skilled and unskilled workers, as well as the amenity level in Detroit, do not change.⁷

Unlike the case of demand, here the nominal wage of skilled workers in San Francisco and Detroit remains unchanged.⁸ Attracted by the better amenity, some skilled workers move from Detroit to San Francisco and some unskilled workers leave San Francisco to Detroit.⁹

⁷For simplicity, I have assumed that supply shocks are driven by increases in amenities for given tastes. Glaeser and Tobio (2007) have a model that makes a similar assumption. Alternatively I could assume that (i) amenities are fixed, but the taste for those amenities increase; or (ii) both amenities and tastes are fixed, but amenities are a normal good so that college graduates consume more of them than high school graduates (Gyourko, Mayer, and Sinai, 2006).

⁸This may be surprising at first. While one might expect wage *increases* in response to demand increases, one might expect wage *decreases* in response to supply increases. Why do nominal wages not decline in San Francisco? The reason is that in a model with capital, nominal wages do not move in San Francisco because capital flows to San Francisco and leaves Detroit, offsetting the changes in labor supply in the two cities. (In a model without capital nominal wages do decline.)

⁹Specifically, the number of skilled workers who move to San Francisco is equal to

On net, the population in San Francisco increases by

$$(N_{Hb2} + N_{Lb2}) - (N_{Hb1} + N_{Lb1}) = \frac{\Delta' N s_L}{h(k_a N (s_H + s_L) + k_b N (s_H + s_L) + 2s_H s_L)} \geq 0 \quad (17)$$

As a consequence, housing costs in San Francisco increase by

$$r_{b2} - r_{b1} = \frac{s_L N k_b \Delta'}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \geq 0 \quad (18)$$

and decline in Detroit by

$$r_{a2} - r_{a1} = -\frac{s_L N k_a \Delta'}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \leq 0 \quad (19)$$

Real wages of skilled workers in San Francisco decline by an amount equal to equation 18 (with a minus sign in front). This reflects the compensating differential for the better amenity in San Francisco. Real wages of skilled workers in Detroit increase by an amount equal to equation 19 (with a minus sign in front).

Similarly, the real wage for unskilled workers in San Francisco declines by

$$(w_{Lb2} - r_{b2}) - (w_{Lb1} - r_{b1}) = -\frac{s_L N k_b}{h(k_a N s_H + 2s_H s_L + k_a N s_L + k_b N s_H + k_b N s_L)} \Delta' \leq 0 \quad (20)$$

and it increases in Detroit.

$\frac{\Delta' N ((k_a + k_b) N + 2s_L)}{2h(k_a N (s_H + s_L) + k_b N (s_H + s_L) + 2s_H s_L)} \geq 0$. The number of unskilled workers who move to Detroit is equal to $\frac{\Delta' N^2 (k_a + k_b)}{2h(k_a N (s_H + s_L) + k_b N (s_H + s_L) + 2s_H s_L)} \geq 0$.

Appendix Table A1. Alternative Measures of Inequality

	1980	2000	1980-2000	1980	2000	1980-2000
	Nominal	Nominal	Nominal	Real	Real	Real
				(CPI 2)	(CPI 2)	(CPI 2)
	(1)	(2)	(3)	(4)	(5)	(6)
<u>Unadjusted Variance</u>						
Total	.280	.346	.066	.278	.352	.074
Within	.196	.261	.065	.191	.262	.071
Bewteen	.084	.087	.003	.086	.090	.004
<u>Adjusted Variance</u>						
Total	.282	.345	.063	.284	.343	.059
Within	.198	.254	.056	.195	.254	.059
Bewteen	.084	.090	.006	.089	.089	.0001
<u>Other (Adjusted) Inequality Measures</u>						
50-10	.561	.610	.049	.543	.606	.063
90-50	.523	.607	.084	.507	.607	.100

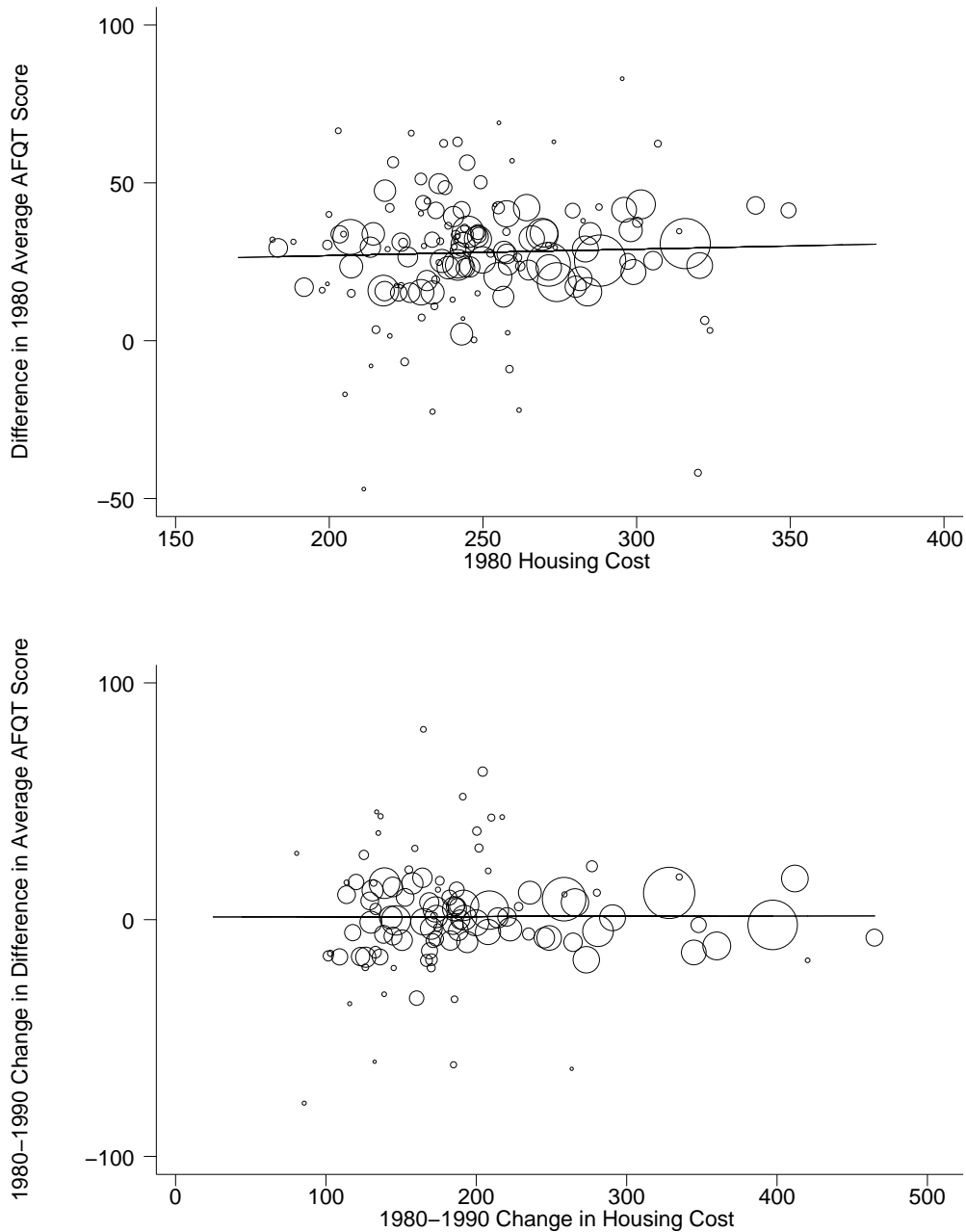
Notes: Standard errors clustered by metropolitan area in parentheses.

Appendix Table 2. Estimates Based on an Alternative Definition of Rental Cost

	1980	1990	2000	1980-2000 Increase	Percent of Nominal Increase Accounted for by Cost of Living
	(1)	(2)	(3)	(4)	(5)
Nominal Wage Difference	.39 (.008)	.53 (.012)	.59 (.013)	.20	
Real Wage - Local CPI 3	.32 (.006)	.41 (.005)	.44 (.004)	.12	40%
Real Wage - Local CPI 4	.28 (.006)	.34 (.006)	.38 (.005)	.10	50%

Notes: Local CPI 3 only uses local variation in cost of living that arises from variation in cost of housing. The difference with Local CPI 1 is that in Local CPI 1 cost of housing varies only by MSA, while in Local CPI 3 cost of housing varies by MSA, education group, race and number of children. Local CPI 4 uses local variation both in cost of housing and cost of non housing good and services. The difference with Local CPI 2 is that in Local CPI 2 cost of housing varies only by MSA, while in Local CPI 4 cost of housing varies by MSA, education group race and number of children. All models include dummies for gender and race, a cubic in potential experience, and year effects. Sample size is 4,920,703.

Figure A1: How the Difference Between College and High-School Graduates Average AFQT Scores Relates to Cost of Housing



Notes: The top panel shows average cost of renting a 2 or a 3 bedroom apartment in 1980 (x-axis) and the difference between college graduates and high school graduates in average AFQT scores (y-axis), across metropolitan areas. The size of the bubbles reflects the size of metropolitan areas. A weighted regression yields a coefficient equal to .0203 (.0274). The bottom panel shows the 1980-1990 change in average cost of renting a 2 or a 3 bedroom apartment (x-axis) and the 1980-1990 change in the difference between college graduates and high school graduates in average AFQT scores (y-axis). A weighted regression yields a coefficient equal to .0010 (.0131).