Online Appendix for "Price, Quality, and Variety: Measuring the Gains from Trade in Differentiated Products"

Gloria Sheu*
US Department of Justice

A Appendix: Theory Details

A.1 NRCL Demand Model

This section explains the theory underlying the NRCL model, with the NL as a special case. The utility for consumer i of type r buying good j in group g is

$$u_{ijt}^r = \ln(a_{jt}^r m_{ijt}^r) + \zeta_{igt}^r + \epsilon_{ijt}^r. \tag{A.1}$$

Here a_{jt}^r is a good-specific measure of quality similar to b_{jt}. The m_{ijt}^r is the quantity of good j that consumer i chooses to buy.

Meanwhile the ζ_{igt}^r is a random draw from a logit distribution with scale parameter μ_1^r, and the ϵ_{ijt}^r is a random draw from a logit distribution with scale parameter μ_2^r.[1] Thus, each consumer has a series of independently and identically distributed (iid) random draws, one for each product $j \in \mathcal{J}_{gt}^r$ and one for each group $g \in \{ink, las\}$.[2]

Those familiar with other applications of the logit model may note that the specification here is slightly different in two ways. These changes were documented by Anderson et al. (1992) as necessary in order the make the MNL align

*The views expressed here are not purported to reflect those of the US Department of Justice. Email: gloria.sheu@usdoj.gov

[1]A random variable x is distributed logit if it has a cumulative distribution function of $\exp\left[-\exp\left(-x/\mu + \varrho\right)\right]$ where μ is the scale parameter and ϱ is Euler's constant (≈ 0.577). This is often referred to as a "Type I Extreme Value" distribution.

[2]Note that here the random component of utility is written as the sum of group- and product-level logit shocks, following Anderson et al. (1992). This facilitates a clear analogy with the two-step choice problem in the NCES. Berry (1994), among others, writes the random component as two terms that sum to a logit shock, governed by a single parameter σ (not to be confused with σ in this paper). The two formulations result in equivalent choice probabilities.

exactly with the CES. First, consumers can buy a continuous amount of their chosen good j, instead of only purchasing of one unit. Second, the non-random part of utility enters in logs rather than in levels. These modifications result in demand equations in terms of expenditure shares (not quantity shares), with price and quality entering multiplicatively (not additively), just as in the NCES.

Each time period, consumer i's problem is to maximize current period utility subject to a budget constraint.[3] The budget constraint is given by $p_{jt}m^r_{ijt} = y^r$ where y^r is the consumer's income. Substituting this constraint into the utility function gives an indirect utility of

$$v^r_{ijt} = \ln(a^r_{jt}) - \ln(p_{jt}) + \ln(y^r) + \zeta^r_{igt} + \epsilon^r_{ijt}.$$

The consumers' problem can be tackled in steps, starting with the demand for goods conditional on being within a certain product group. When focusing on one group, the ζ^r_{igt} term drops out. Integrating over the remaining logit random shocks gives,

$$prob^r_{jt|g} = \frac{{a^r_{jt}}^{\frac{1}{\mu^r_2}} p_{jt}^{\frac{-1}{\mu^r_2}}}{\sum_{j \in \mathcal{J}^r_{gt}} {a^r_{jt}}^{\frac{1}{\mu^r_2}} p_{jt}^{\frac{-1}{\mu^r_2}}},$$

which is the conditional probability that any type r consumer will choose good j.

Turning to the choice of which product group to buy from, the consumer chooses the group with the maximum expected indirect utility, which results in a group probability of

$$prob^r_{gt} = \frac{\left(\sum_{j \in \mathcal{J}^r_{gt}} {a^r_{jt}}^{\frac{1}{\mu^r_2}} p_{jt}^{\frac{-1}{\mu^r_2}}\right)^{\frac{\mu^r_2}{\mu^r_1}}}{\sum_{g \in \{ink, las\}} \left(\sum_{j \in \mathcal{J}^r_{gt}} {a^r_{jt}}^{\frac{1}{\mu^r_2}} p_{jt}^{\frac{-1}{\mu^r_2}}\right)^{\frac{\mu^r_2}{\mu^r_1}}}.$$

Following Anderson et al. (1992), let ${a^r_{jt}}^{1/\mu^r_2} = b^r_{jt}$, $-1/\mu^r_2 = 1 - \sigma^r$, and $1/\mu^r_1 = \gamma^r - 1$. Then convert $prob^r_{jt|g}$ and $prob^r_{gt}$ to (expected) expenditure shares by multiplying and dividing by the consumer's income. The resulting expenditure shares are

$$s^r_{jt|g} = \frac{b^r_{jt} p_{jt}^{1-\sigma^r}}{\sum_{j \in \mathcal{J}^r_{gt}} b^r_{jt} p_{jt}^{1-\sigma^r}} \tag{A.2}$$

[3]This is an entirely static problem, with no borrowing or saving.

and

$$s_{gt}^{r} = \frac{\left(\sum_{j \in \mathcal{J}_{gt}^{r}} b_{jt}^{r} p_{jt}^{1-\sigma^{r}}\right)^{\frac{1-\gamma^{r}}{1-\sigma^{r}}}}{\sum_{g \in \{ink,las\}} \left(\sum_{j \in \mathcal{J}_{gt}^{r}} b_{jt}^{r} p_{jt}^{1-\sigma^{r}}\right)^{\frac{1-\gamma^{r}}{1-\sigma^{r}}}}. \tag{A.3}$$

Multiplying these two shares gives

$$s_{jt}^{r} = \frac{b_{jt}^{r} p_{jt}^{1-\sigma^{r}}}{\left(\sum_{j \in \mathcal{J}_{gt}^{r}} b_{jt}^{r} p_{jt}^{1-\sigma^{r}}\right)^{\frac{\gamma^{r}-\sigma^{r}}{1-\sigma^{r}}} \sum_{g \in \{ink,las\}} \left(\sum_{j \in \mathcal{J}_{gt}^{r}} b_{jt}^{r} p_{jt}^{1-\sigma^{r}}\right)^{\frac{1-\gamma^{r}}{1-\sigma^{r}}}}. \tag{A.4}$$

If all types of consumers have identical preferences, meaning that $b_{jt}^{r} = b_{jt}$, $\sigma^{r} = \sigma$, and $\gamma^{r} = \gamma$ for all r, these formulas collapse down to those in the NL model. If in turn $\sigma = \gamma$, the model reduces to the MNL.

The market-level share is found by integrating s_{jt}^{r} across the distribution of consumer types. For example, if the distribution is discrete, the share is then

$$s_{jt} = \sum_{r \in \{sm,lg\}} f_{t}^{r} s_{jt}^{r}, \tag{A.5}$$

where f_{t}^{r} is the fraction of expenditure accounted for by type r consumers in time t.

A.2 NCES Demand Model

This section shows how the NCES and NL are related. Assume there is a representative consumer that has a utility function given by

$$U_{t} = \left(\sum_{g \in \{ink,las\}} M_{gt}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}, \quad \text{where} \quad \gamma > 1. \tag{A.6}$$

The consumption of each group is denoted by M_{gt}. Within each product group g the consumer has an inner nested utility function of the form

$$M_{gt} = \left(\sum_{j \in \mathcal{J}_{gt}} b_{jt}^{\frac{1}{\sigma}} m_{jt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \quad \text{where} \quad \sigma > 1. \tag{A.7}$$

Each time period the consumer's problem is to maximize current period utility subject to a budget constraint. The utility maximization problem can be solved in two stages. First, maximize M_{gt} conditional on the amount of money allocated to group g. Then decide on the allocation of expenditure across groups. This exercise results in the expression for the share of expenditure allocated to product j within group g,

$$s_{jt|g} = \frac{b_{jt} p_{jt}^{1-\sigma}}{\sum_{j \in \mathcal{J}_{gt}} b_{jt} p_{jt}^{1-\sigma}}. \tag{A.8}$$

In turn, the share of expenditure devoted to group g out of total expenditure is

$$s_{gt} = \frac{\left(\sum_{j \in \mathcal{J}_{gt}} b_{jt} p_{jt}^{1-\sigma} \right)^{\frac{1-\gamma}{1-\sigma}}}{\sum_{g \in \{ink,las\}} \left(\sum_{j \in \mathcal{J}_{gt}} b_{jt} p_{jt}^{1-\sigma} \right)^{\frac{1-\gamma}{1-\sigma}}}. \tag{A.9}$$

Multiplying these two expressions gives the share of expenditure allocated to product j out of the money spent on all product groups,

$$s_{jt} = \frac{b_{jt} p_{jt}^{1-\sigma}}{\left(\sum_{j \in \mathcal{J}_{gt}} b_{jt} p_{jt}^{1-\sigma} \right)^{\frac{\gamma-\sigma}{1-\sigma}} \sum_{g \in \{ink,las\}} \left(\sum_{j \in \mathcal{J}_{gt}} b_{jt} p_{jt}^{1-\sigma} \right)^{\frac{1-\gamma}{1-\sigma}}}. \tag{A.10}$$

These are the same expenditure share equations that appear in the NL model. In the special case where $\sigma = \gamma$, the NCES reduces to the CES model.

B Appendix: Additional Data Details

Summary statistics for the printer data, including model characteristics, are in Table B.1. IDC categorizes each model into one of several groups based on the type of printer (multi-function or single function), technology (inkjet, laser), color versus monotone printing, and print speed. These categorizations provide the groupings used to construct the exchange rate instrument and are the basis for the aggregations used in the NCES results. The categories cross-referenced with the headquarters countries that appear in each are listed in Table B.2.

References

Anderson, Simon P., André de Palma, and Jacques-François Thisse, *Discrete Choice Theory of Product Differentiation*, MIT Press, 1992. [1, 2]

Berry, Steven, "Estimating Discrete-Choice Models of Product Differentiation," *RAND Journal of Economics*, 1994, *25* (2), 242–262. [1]

Table B.1: Summary Statistics

Variable	Mean	Standard Deviation
Price (USD)	604.996	1084.060
Units Sold	1059.538	4763.041
Color Dummy	0.468	0.499
BW PPM Speed	20.806	13.430
RAM (MB)	49.764	98.272
Resolution (DPI)	1336.798	771.611
A3 Capable Dummy	0.355	0.478
Footprint (in^2)	416.175	381.911
Ethernet Interface Dummy	0.343	0.475
MFP Dummy	0.356	0.479
Laser Dummy	0.663	0.473
Number of Model-Quarters	6413	
Number of Unique Models	1189	

Notes: Data sources are in the main text of the paper, Section II. Price is in real 2001 Indian Rs, then converted to USD at 1 Rs=47.12 USD. "BW PPM Speed" is the maximum number of pages per minute that can be printed in black and white.

Table B.2: Product Categories

Product Type	Japan	US	Korea	EU
MFP Color Inkjet 1-10 PPM	X	X	X	
MFP Color Inkjet 11-20 PPM	X	X	X	
MFP Color Inkjet 21 PPM or more	X	X		
MFP Color Laser 1-10 PPM	X	X		
MFP Color Laser 11-20 PPM	X	X		
MFP Color Laser 21-30 PPM	X	X		
MFP Color Laser 31-44 PPM	X	X		X
MFP Mono Inkjet All Speeds			X	
MFP Mono Laser 1-20 PPM	X	X	X	
MFP Mono Laser 21-30 PPM	X	X	X	
MFP Mono Laser 31-44 PPM	X	X		
MFP Mono Laser 45-69 PPM	X	X		X
MFP Mono Laser 70-90 PPM	X	X		
Printer Color Inkjet 1-10 PPM	X	X		
Printer Color Inkjet 11-20 PPM	X	X		
Printer Color Inkjet 21 PPM or more	X	X		
Printer Color Laser 1-10 PPM	X	X	X	
Printer Color Laser 11-20 PPM	X	X		
Printer Color Laser 21-30 PPM	X	X		
Printer Color Laser 31-44 PPM		X		
Printer Mono Inkjet All Speeds	X	X		
Printer Mono Laser 1-20 PPM	X	X	X	
Printer Mono Laser 21-30 PPM	X	X	X	
Printer Mono Laser 31-44 PPM	X	X		
Printer Mono Laser 45-69 PPM	X	X		
Printer Mono Laser 70-90 PPM		X		

Notes: Product types are from the IDC taxonomy. “PPM” stands for pages per minute.