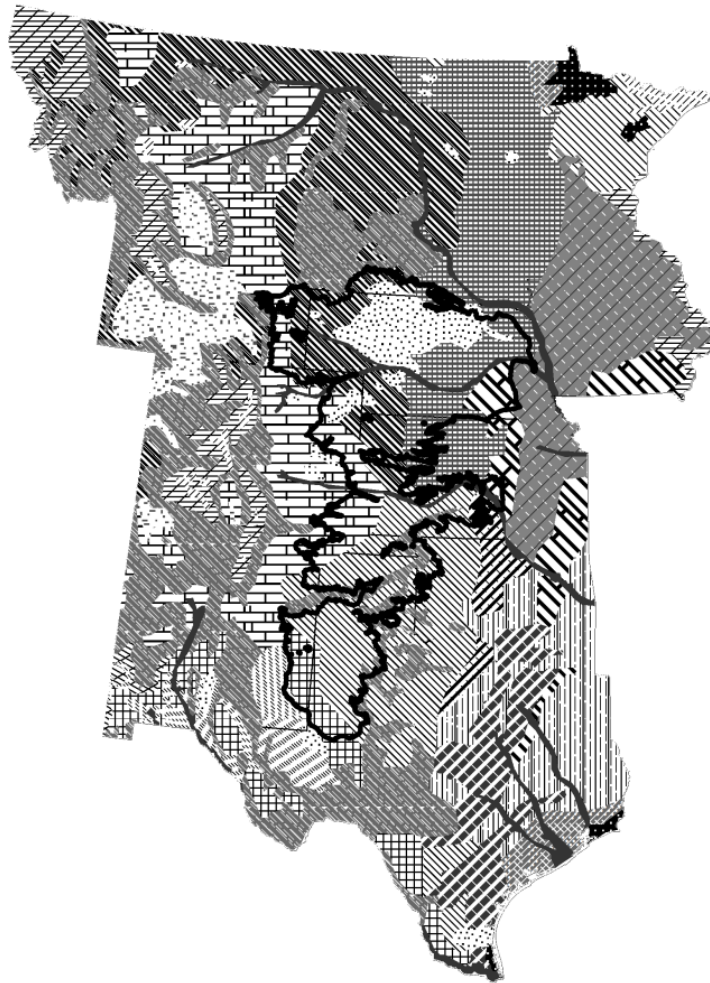# The Historically Evolving Impact of the Ogallala Aquifer: Agricultural Adaptation to Groundwater and Drought

Richard Hornbeck and Pinar Keskin

Online Appendices: Figures, Tables, Proofs

Appendix Figure 1. Ogallala Boundary and Soil Group Control Variables

Soil Groups appearing within Ogallala

- Alluval Soils
- Brown Soils
- Chernozem Soils
- Chestnut Soils
- Lithosols & Shallow Soils - Arid Subhumid
- Planosols
- Podzol Soils
- Red Desert Soils
- Reddish Brown Soils
- Reddish Chestnut Soils
- S&s (Dry)
- Sierozem Brown Soils

Other Soil Groups

- Bog Soils
- Gray - Brown Podzolic Soils
- Prairie Soils
- Red & Yellow Podzolic Soils
- Lithosols & Shallow Soils - Humid
- Noncalcic Brown Soils
- Reddish Soils
- Rendzina Soils
- Wiesenböden & Ground Water Podzol & Half-Bog Soils

Notes: The Ogallala boundary (USGS) is overlaid with major soil groups, as mapped by the Soil Conservation Service (SCS 1951).

Appendix Figure 2. Ogallala Boundary and Natural Vegetation Regions

Vegetation Groups appearing within Ogallala
- Desert savana
- Oak-Hickory
- Short grass
- Tall Grass
- Yellow pine-sugar pine

Other Vegetation Groups
- Birch Beech Maple Hemlock
- Bunch Grass
- Cedar Hemlock Wetsren Larch -Western White Pine
- Creosote bush
- Cypress Tupelo Red Gum
- Greasewood
- Jack,red & white pines
- Lodgepole Pine
- Long Leaf Lobolly Slash Pines
- Mesquite Grassland Desert grass
- Oak-Pine
- Pinon-Juniper
- Sagebrush
- Spruce fir
- Woodland
- Yellow pine

Notes: The Ogallala boundary (USGS) is overlaid with natural vegetation regions, as mapped by the 1924 Atlas of Agriculture (USDA 1924).

Appendix Table 1. Estimated Local Spillover Impacts: Nearby Non-Ogallala Counties vs. Counties 100km from the Ogallala

	Irrigated Farmland	Farmland	Corn Acres Harvested		Wheat Acres Harvested		Log Farm Value	Log Farm Revenue
			Irrigated Corn	All Corn	Irrigated Wheat	All Wheat		
Coefficient in era:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1920-1945	0.001	0.066	-0.0000	0.021	-0.0004	0.023	0.259	0.136
	(0.006)	(0.044)	(0.0004)	(0.014)	(0.0009)	(0.016)	(0.182)	(0.198)
1950-1974	-0.002	-0.016	0.0038	0.013	-0.0003	-0.002	0.037	0.008
	(0.010)	(0.048)	(0.0027)	(0.016)	(0.0002)	(0.017)	(0.129)	(0.210)
1978-2002	0.002	0.035	0.0125	0.003	-0.0005	-0.010	0.068	0.150
	(0.012)	(0.051)	(0.0079)	(0.013)	(0.0013)	(0.021)	(0.111)	(0.253)
Sample Counties	136	136	114	133	99	135	136	136

Notes: For counties with zero area over the Ogallala, each column reports estimates from a modified version of equation (3) in the text. The coefficients report the impact of "Negative Distance to Ogallala Boundary," measured in 100km units, which reflects average outcomes in counties next to the Ogallala boundary relative to counties 100km away. Otherwise, the specifications are as described in Tables 2-4. Data for some outcome variables are only available in some years (shown in Figures 5-8). Robust standard errors clustered by county are reported in parentheses. ** denotes statistical significance at the 1% level, * at the 5% level.

I Theory Appendix

This appendix contains proofs of the theoretical results discussed in section II.

I.A Model Setup

The maximization problem of our representative farmer is

$$\max_{L_1,L_2,w_1,w_2} y_1(L_1, w_1, d) + y_2(L_2, w_2, d)$$

subject to the constraints

$$
\begin{aligned}
w_1 + w_2 &= \bar{w} \\
L_1 + L_2 &= \bar{L}
\end{aligned}
$$

The production functions are globally concave, with five additional assumptions:

1. The marginal product of water is higher for the first crop:

    $\partial y_1/\partial w_1 > \partial y_2/\partial w_2 > 0.$

2. The marginal product of water declines slower for the first crop:

    $\partial^2 y_2/(\partial w_2)^2 < \partial^2 y_1/(\partial w_1)^2 < 0.$

3. Water and land are complementary for both crops, but weakly more so for the first crop:

    $\partial^2 y_1/\partial L_1 \partial w_1 \geq \partial^2 y_2/\partial L_2 \partial w_2 > 0.$

4. Drought decreases the productivity of land for both crops, but drought has a larger negative effect on the water-intensive crop:

    $\partial^2 y_1/\partial L_1 \partial d < \partial^2 y_2/\partial L_2 \partial d < 0.$

5. Drought increases the productivity of water for both crops, but more so for the water-intensive crop:

    $\partial^2 y_1/\partial w_1 \partial d > \partial^2 y_2/\partial w_2 \partial d > 0.$

I.B   Comparative Statics Without Drought

Initially, suppress the impact of drought (d) on production. The first order conditions for the farmer's maximization problem are given by:

$$\frac{\partial y_1\left(L_1^*, w_1^*\right)}{\partial w_1} - \frac{\partial y_2\left(\bar{L} - L_1^*, \bar{w} - w_1^*\right)}{\partial w_2} = 0$$

$$\frac{\partial y_1\left(L_1^*, w_1^*\right)}{\partial L_1} - \frac{\partial y_2\left(\bar{L} - L_1^*, \bar{w} - w_1^*\right)}{\partial L_2} = 0.$$

Of interest is how optimal factor allocation responds to a change in the available water $(\bar{w})$.

Proposition 1. *Water and land allocated to the water intensive crop are increasing in total water availability.*

Proof Totally differentiating the first order conditions with respect to $\bar{w}$, we obtain

$$\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1}\frac{\partial L_1^*}{\partial \bar{w}} + \frac{\partial^2 y_1(\cdot)}{(\partial w_1)^2}\frac{\partial w_1^*}{\partial \bar{w}} - \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}\left(1 - \frac{\partial w_1^*}{\partial \bar{w}}\right) + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\frac{\partial L_1^*}{\partial \bar{w}} = 0, \text{ and}$$

$$\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1}\frac{\partial w_1^*}{\partial \bar{w}} + \frac{\partial^2 y_1(\cdot)}{(\partial L_1)^2}\frac{\partial L_1^*}{\partial \bar{w}} + \frac{\partial^2 y_2(\cdot)}{(\partial L_2)^2}\frac{\partial L_1^*}{\partial \bar{w}} - \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\left(1 - \frac{\partial w_1^*}{\partial \bar{w}}\right) = 0.$$

Rewriting, we obtain

$$[\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}]\frac{\partial L_1^*}{\partial \bar{w}} + [\frac{\partial^2 y_1(\cdot)}{(\partial w_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}]\frac{\partial w_1^*}{\partial \bar{w}} = \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}, \text{ and}$$

$$[\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}]\frac{\partial w_1^*}{\partial \bar{w}} + [\frac{\partial^2 y_1(\cdot)}{(\partial L_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial L_2)^2}]\frac{\partial L_1^*}{\partial \bar{w}} = \frac{\partial^2 y_2(\cdot)}{\partial w_2 \partial L_2}.$$

The solution to this system is

$$\frac{\partial L_1^*}{\partial \bar{w}} = \frac{\left(\frac{\partial^2 y_1(\cdot)}{(\partial w_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}\right)\frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2} - \left(\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\right)\frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}}{\left(\frac{\partial^2 y_1(\cdot)}{(\partial w_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}\right)\left(\frac{\partial^2 y_1(\cdot)}{(\partial L_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial L_2)^2}\right) - \left(\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\right)^2}$$

$$\frac{\partial w_1^*}{\partial \bar{w}} = \frac{\left(\frac{\partial^2 y_1(\cdot)}{(\partial L_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial L_2)^2}\right)\frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2} - \left(\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\right)\frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}}{\left(\frac{\partial^2 y_1(\cdot)}{(\partial w_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial w_2)^2}\right)\left(\frac{\partial^2 y_1(\cdot)}{(\partial L_1)^2} + \frac{\partial^2 y_2(\cdot)}{(\partial L_2)^2}\right) - \left(\frac{\partial^2 y_1(\cdot)}{\partial w_1 \partial L_1} + \frac{\partial y_2(\cdot)}{\partial w_2 \partial L_2}\right)^2}$$

Global concavity of the revenue function $(y_1 + y_2)$ ensures that the denominators in $\partial L_1^*/\partial \bar{w}$ and $\partial w_1^*/\partial \bar{w}$ are positive. Under assumptions 1 - 3, above, the numerators are also positive. Thus, $\partial L_1^*/\partial \bar{w} > 0$ and $\partial w_1^*/\partial \bar{w} > 0$.

I.C   General Case: Comparative Statics With Drought

Proposition 2. *When the land allocation is held constant, an increase in water availability reduces the (negative) impact of drought:*

$$\frac{d}{d\bar{w}} \left[\frac{\partial y_1 \left(L_1^*, w_1^* \left(\bar{w} \right), d \right)}{\partial d} + \frac{\partial y_2 \left(L_2^*, w_2^* \left(\bar{w} \right), d \right)}{\partial d} \right] > 0.$$

Conversely, when the land allocation can respond to changes in $\bar{w}$, an increase in water availability has an ambiguous effect on the impact of drought.

Proof Consider the effect of $\bar{w}$ on the derivative of the revenue function with respect to d.

$$\begin{aligned}
\frac{d}{d\bar{w}} \left[\frac{\partial y_1}{\partial d} + \frac{\partial y_2}{\partial d} \right] &= \frac{\partial^2 y_1}{\partial d \partial L_1} \frac{\partial L_1^*}{\partial \bar{w}} + \frac{\partial^2 y_1}{\partial d \partial w_1} \frac{\partial w_1^*}{\partial \bar{w}} - \frac{\partial^2 y_2}{\partial d \partial L_2} \frac{\partial L_1^*}{\partial \bar{w}} + \frac{\partial^2 y_2}{\partial d \partial w_2} \frac{\partial w_2^*}{\partial \bar{w}} \\
&= \underbrace{\left(\frac{\partial^2 y_1}{\partial d \partial w_1} \frac{\partial w_1^*}{\partial \bar{w}} + \frac{\partial^2 y_2}{\partial d \partial w_2} \frac{\partial w_2^*}{\partial \bar{w}} \right)}_{>0} + \underbrace{\left(\frac{\partial^2 y_1}{\partial d \partial L_1} - \frac{\partial^2 y_2}{\partial d \partial L_2} \right) \frac{\partial L_1^*}{\partial \bar{w}}}_{<0}
\end{aligned}$$

Assumption 5 and Proposition 1 imply that the first term is positive. Assumption 4 and Proposition 1 imply that the second term is negative. Thus, an increase in water availability has an ambiguous effect on the impact of drought. If the land allocation is held fixed $(\partial L_1^* / \partial \bar{w} = 0)$, then the impact is unambiguously positive. In addition, the effect of water availability on the impact of drought is more positive than when the land allocation is free to adjust.

I.D   Special Case: Comparative Statics With Drought

Consider the special case of constant returns to land, in which the farmer maximizes

$$L_1 y_1(w_1, d) + L_2 y_2(w_2, d).$$

Proposition 3. *When the production technology displays constant returns to land:*

1. *If the land allocation can adjust to $\bar{w}$, then an increase in water availability increases the (negative) impact of drought:*

$$\frac{d}{d\bar{w}} \left[L_1^* \left(\bar{w} \right) \frac{\partial y_1 \left(w_1^* \left(\bar{w} \right), d \right)}{\partial d} + L_2^* \left(\bar{w} \right) \frac{\partial y_2 \left(w_2^* \left(\bar{w} \right), d \right)}{\partial d} \right] < 0.$$

2. *If the land allocation is fixed, then an increase in water availability reduces the (negative) impact of drought.*

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Proof The first order conditions simply to:

$$\frac{\partial y_1(w_1, d)}{\partial w_1} = \frac{\partial y_2(w_2, d)}{\partial w_2}$$

$$y_1(w_1, d) - y_2(w_2, d) = \frac{\partial y_2(w_2, d)}{\partial w_2}(w_1 - w_2).$$

Assumption 1 and global concavity imply that

$$w_1^* > w_2^*.$$

In deriving the impact of $\bar{w}$, total differentiation of the first order conditions yields:

$$\frac{\partial^2 y_1(w_1, d)}{(\partial w_1)^2}\frac{\partial w_1}{\partial \bar{w}} = \frac{\partial^2 y_2(w_2, d)}{(\partial w_2)^2}\frac{\partial w_2}{\partial \bar{w}}$$

$$\frac{\partial y_1(w_1, d)}{\partial w_1}\frac{\partial w_1}{\partial \bar{w}} - \frac{\partial y_2(w_2, d)}{\partial w_2}\frac{\partial w_2}{\partial \bar{w}} = \frac{\partial^2 y_2(w_2, d)}{(\partial w_2)^2}\frac{\partial w_2}{\partial \bar{w}}(w_1 - w_2) + \frac{\partial y_2(w_2, d)}{\partial w_2}\left(\frac{\partial w_1}{\partial \bar{w}} - \frac{\partial w_2}{\partial \bar{w}}\right).$$

Using the first order conditions to simply the second expression:

$$\frac{\partial^2 y_2(w_2, d)}{(\partial w_2)^2}\frac{\partial w_2}{\partial \bar{w}}(w_1 - w_2) = 0.$$

Because $w_1^* \neq w_2^*$, the only solution is $\partial w_1/\partial \bar{w} = \partial w_2/\partial \bar{w} = 0$. That is, increased water availability does not cause the farmer to use more water per acre; instead, the farmer shifts land toward the more water-intensive crop. Because w_1 and w_2 are constants,

$$L_1^*(\bar{w}) = \frac{\bar{w} - w_2}{w_1 - w_2},$$

and L_1^* is increasing in $\bar{w}$. Substituting this special case into the general solution:

$$\frac{\mathrm{d}}{\mathrm{d}\bar{w}}[L_1^*(\bar{w})\frac{\partial y_1}{\partial d} + L_2^*(\bar{w})\frac{\partial y_2}{\partial d}] = \frac{\partial L_1^*}{\partial \bar{w}}\frac{\partial y_1}{\partial d} + L_1^*\frac{\partial^2 y_1}{\partial d\partial w_1}\frac{\partial w_1^*}{\partial \bar{w}} + \frac{\partial L_2^*}{\partial \bar{w}}\frac{\partial y_2}{\partial d} + L_2^*\frac{\partial^2 y_2}{\partial d\partial w_2}\frac{\partial w_2^*}{\partial \bar{w}}$$

$$= \underbrace{\left(L_1^*\frac{\partial^2 y_1}{\partial d\partial w_1}\frac{\partial w_1^*}{\partial \bar{w}} + L_2^*\frac{\partial^2 y_2}{\partial d\partial w_2}\frac{\partial w_2^*}{\partial \bar{w}}\right)}_{=0} + \underbrace{\left(\frac{\partial y_1}{\partial d} - \frac{\partial y_2}{\partial d}\right)\frac{\partial L_1^*}{\partial \bar{w}}}_{<0}.$$

Thus, when land allocations can adjust and the production technology displays constant returns to land, an increase in water availability increases the (negative) impact of drought. If the land allocation is fixed, however, increased water availability can only be allocated on the intensive margin and the (negative) impact of drought declines.